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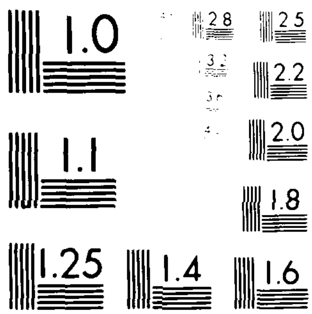

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COMPUTER PROGRAM FOR ELECTROMAGNETIC COUPLING TO A CONDUCTING BODY OF REVOLUTION WITH A HOMOGENEOUS MATERIAL REGION

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A computer program is given to calculate electromagnetic coupling to a perfectly conducting body of revolution with a loss-free homogeneous material region. The material region is also a body of revolution. It is bounded by an aperture and part of the surface of the conducting body. The maximum dimensions of the conducting body and its associated material region are of the order of a few wavelengths. The electromagnetic excitation is an obliquely incident θ-polarized plane wave.

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Application of the equivalence principle and subsequent enforcement of the boundary conditions for tangential fields give a set of integral equations. These equations are then solved by means of the method of moments. The computer program calculates the electric current induced on the surface of the conducting body and the equivalent electric and magnetic currents in the aperture. These currents radiate the field scattered by the conducting body and its associated material region. They also radiate the field transmitted through the aperture into the material region. The computer program is described and listed along with sample input and output data.

CONTENTS

I. INTRODUCTION

The computer program presented here calculates the Fourier coefficients [1, Eqs. (87)-(89)] of the electric and magnetic currents on the object shown in [1 , Fig. 1] excited by an obliquely incident θ-polarized plane wave. The I's and V's in [1, Eqs. (87)-(89)] are the elements [1, Eq. (68a)] of $\tilde{X}_n^\theta$ obtained by solving [1, Eq. (65a)] for non-negative values of n with $\tilde{B}_n^\theta$ given by [1, Eq. (101)].

The computer program consists of a main program and the subprograms YZA, BLOG, PLANE, DECOMP, SOLVE, and PRNT. The subroutine YZA calculates the matrix elements [1, Eqs. (35) and (36)] which appear in expression [1, Eq. (33)] for the moment matrix T_n. T_n is needed in [1, Eq. (65a)]. The subroutine YZA calls the function BLOG. The subroutine PLANE calculates $V_{ni}^{t\theta}$', $-V_{ni}^{\phi\theta}$', $-V_{ni}^{t\phi}$', and $V_{ni}^{\phi\phi}$', which, according to [1, Eq. (103)], determine $\tilde{B}_n^\theta$ of [1, Eq. (101)]. The subroutine DECOMP decomposes T_n into the product of a lower triangular matrix with an upper triangular matrix. The subroutine SOLVE uses these triangular matrices to calculate the solution $\tilde{X}_n^\theta$ to [1, Eq. (65a)]. The subroutine PRNT uses the elements of $\tilde{X}_n^\theta$ of [1, Eq. (68a)] to calculate the Fourier coefficients [1, Eqs. (87)-(89)]. The subroutine PRNT also prints out [1, Eqs. (87)-(89)].

The main program calls the subroutines YZA, PLANE, DECOMP, SOLVE, and PRNT in order to calculate and print out [1, Eqs. (87)-(89)]. The main program is rather long because it has to rearrange the Z's and Y's calculated by the subroutine YZA and the V's calculated by the subroutine PLANE. Unfortunately, the Z's and Y's come out of the subroutine YZA arranged not as in [1, Eqs. (35) and (36)] but as on the right-hand sides of [1, Eqs. (47) and (50)] for consecutive values of i' and j'. Likewise, the V's exit the subroutine PLANE arranged not as on the left-hand sides of [1, Eq. (103)] but as on the right-hand sides of [1, Eq. (103)] for consecutive values of i'. The main program has to rearrange these Z's, Y's, and V's so as to realize the transition from $\underline{W}^{r\pm}_{ni'}$ and $\underline{J}^{s\pm}_{nj'}$ to $\underline{W}^{p\pm}_{ni}$ and $\underline{J}^{q\pm}_{nj}$.

II. THE SUBROUTINE YZA

The subroutine YZA(M1, M2, NP, NPHI, NT, IN, RH, ZH, X, A, XT, AT, Y, Z) calculates the elements of matrices Y'_n and Z_n defined by

$$Y'_n = \begin{bmatrix} Y'^{tt}_n & Y'^{t\phi}_n \\ Y'^{\phi t}_n & Y'^{\phi\phi}_n \end{bmatrix}, \quad n=M1,\ M1+1,\ldots M2 \tag{1}$$

$$Z_n = \begin{bmatrix} Z^{tt}_n & Z^{t\phi}_n \\ Z^{\phi t}_n & Z^{\phi\phi}_n \end{bmatrix}, \quad n=M1,\ M1+1,\ldots,M2 \tag{2}$$

where the ijth elements of the submatrices on the right-hand sides of (1) and (2) are given by

$$\left.\begin{aligned} Y'^{rs}_{nij} &= -\langle \underline{W}^r_{ni},\ H(\underline{J}^s_{nj},\ 0)\rangle & (3) \\ Z^{rs}_{nij} &= -\langle \underline{W}^r_{ni},\ \frac{1}{\eta} E(\underline{J}^s_{nj},\ 0)\rangle & (4) \end{aligned}\right\} \quad \begin{aligned} r &= t,\phi \\ s &= t,\phi \end{aligned}$$

The quantities on the right-hand sides of (3) and (4) are those on the right-hand sides of [1, Eqs. (50) and (47)] with the ± notation omitted and with i' and j' replaced by i and j. The < > notation in (3) and (4) denotes the same symmetric product as in [1, Eqs. (50) and (47)]. The ± notation in [1] serves to distinguish the surface $(S^+ + A)$ from the surface $(S^- + A)$. The ± notation is not needed in (3) and (4) because the subroutine YZA deals with only one surface of revolution at a time. Later on in Section V, the main program calls YZA twice, once for $(S^+ + A)$ and once for $(S^- + A)$. For a particular value of n, Y'_n of (1) is stored by columns in Y and Z_n of (2) is stored by columns in Z. Y'_{n+1} is placed immediately after Y'_n in Y. Similarly, Z_{n+1} follows Z_n in Z. Y and Z are the only output arguments of YZA. The rest of the arguments of YZA are input arguments.

The input arguments of YZA have the same meaning as those of the subroutine YZ presented in [2, pages 65-79]. In comparison with YZA, the subroutine YZ of [2] puts Y_n in Y and Z_n of (2) in Z where

$$Y_n = \begin{bmatrix} Y_n^{tt} & Y_n^{t\phi} \\ Y_n^{\phi t} & Y_n^{\phi\phi} \end{bmatrix} \tag{5}$$

The ijth elements of the submatrices on the right-hand side of (5) are given by

$$Y_{nij}^{rs} = -\langle \underline{W}_{ni}^r, \underline{n} \times \underline{H}(\underline{J}_{nj}^s, 0\rangle \qquad \begin{cases} r = t,\phi \\ s = t,\phi \end{cases} \tag{6}$$

where [2, Eq. (4)]

$$\underline{n} = \underline{u}_\phi \times \underline{u}_t \tag{7}$$

As defined by (7), $\underline{n}$ is a unit vector normal to the surface of revolution.

Although we have been omitting the vector designation from vectors inside the symmetric product, we decided to designate $\underline{n}$ and $\underline{H}$ as vectors in (6) to clearly indicate that the vector product $\underline{n} \times \underline{H}$ is intended there.

YZ of [2] allows for the Ampere's law contribution to the magnetic field $\underline{H}$ in (6). The Ampere's law contribution to the magnetic field is the contribution due to the value of the electric current at the field point. The Ampere's law contribution in the subroutine YZ of [2] is controlled by means of the input argument IN of YZ of [2]. The action of IN is described in [2, page 39] under the assumptions that

1) $\underline{n}$ is given by (7)
2) $\underline{n}$ points outward from the surface of revolution.

However, the action of IN can be described without recourse to assumption 2) in the following manner.

$$\left.\begin{array}{ll} \text{IN} = 1, & \text{Magnetic field evaluation on the side of the} \\ & \text{electric current sheet where the } \underline{\text{tail}} \text{ of } \underline{n} \text{ is.} \\ \text{IN} = 0, & \text{No Ampere's law contribution.} \\ \text{IN} = -1, & \text{Magnetic field evaluation on the side of the} \\ & \text{electric current sheet where the } \underline{\text{head}} \text{ of } \underline{n} \text{ is.} \end{array}\right\} \tag{8}$$

In (8), $\underline{n}$ is visualized as piercing the sheet of electric current which produces the magnetic field. Apparently, assumption 2) for $\underline{n}$ can be eliminated from [2] by replacing the description [2, page 39] of the action of the input argument IN of YZ of [2] by (8).

Presumably, the magnetic field $\underline{H}$ in (3) contains no Ampere's law contribution because the magnetic field $\underline{H}^{\pm}$ in [1, Eq. (50)] had no Ampere's law contribution. However, we decided to allow for the Ampere's law contribution associated with $\underline{H}$ in (3) in order to enhance the usefulness of YZA. This Ampere's law contribution is controlled by means of the input argument IN of YZA. The action of IN is described by (8) where it is assumed that $\underline{n}$ is given by (7). We do not require $\underline{n}$ to point outward in (8).

Most of the statements in YZA are exactly the same as those in YZ of [2]. In the remaining part of this section, the differences between YZA and YZ of [2] are pointed out and explained and then YZA is listed. Unless stated otherwise, all line numbers and statement numbers cited henceforth in this section refer to the listing of YZA.

In line 4, more space is allotted to Y and Z than in YZ of [2]. Nevertheless, the minimum allocations given in [2, pages 65-66] apply to YZA. Line 9 in YZ of [2] has no counterpart in YZA because UG and UH are not used in YZA. On the other hand, line 17 has no counterpart in YZ of [2]. Line 17 enables YZA to calculate the moment matrix for the E-field solution [3] for the two conducting bodies in [1, Fig. 13]. This E-field solution was obtained by connecting the generating curves ABC and DEF in [1, Fig. 13] to obtain the single curve ABCDEF and then deleting all matrix elements associated with the connecting line CD. Line 17 sets $k\rho = 1$ at the midpoint of the connecting line CD. Here, k is the propagation constant and ρ is the distance from the z axis. If line 17 were absent, this $k\rho$ would be zero and divisions by zero would occur. The value 1 assigned to $k\rho$ in line 17 is not critical because, as stated earlier, all matrix elements associated with the connecting line CD are deleted. Line 17 also enables YZA to calculate the moment matrix for an H-field solution for the two conducting bodies in [1, Fig. 13]. However, this H-field solution, being a solution to [2, Eq. (1)] without the $\underline{n}\times$, would fail according to the discussion in the third from the last paragraph in [1, Section III]. The H-field solution shown in [1, Fig. 15] was obtained by using the subroutine YZ of [2] modified by setting $k\rho = 1$ at the midpoint of the connecting line CD in [1, Fig. 13].

The major difference between YZ of [2] and YZA lies in the fact that there is an $\underline{n}\times$ in (6) for Y^{rs}_{nij} calculated by YZ of [2] but not in (3) for Y'^{rs}_{nij} calculated by YZA. Because $\underline{W}^{r}_{ni}$ in (3) is a tangential vector, (3) can be written as

$$Y'^{rs}_{nij} = -\langle \underline{n} \times \underline{W}^{r}_{ni}, \underline{n} \times \underline{H}(\underline{J}^{s}_{nj}, 0)\rangle \quad \begin{cases} r = t,\phi \\ s = t,\phi \end{cases} \tag{9}$$

In view of (7) and the assumptions that $\underline{W}^{t}_{ni}$ has only a t component and $\underline{W}^{\phi}_{ni}$ has only a ϕ component, (9) becomes

$$\left.\begin{aligned} Y'^{ts}_{nij} &= -\hat{Y}^{\phi s}_{nij} && \text{(10a)} \\ Y'^{\phi s}_{nij} &= \hat{Y}^{ts}_{nij} && \text{(10b)} \end{aligned}\right\} \quad s = t,\phi$$

where

$$\hat{Y}^{rs}_{nij} = -\langle \underline{\hat{W}}^{r}_{ni}, \underline{n} \times \underline{H}(\underline{J}^{s}_{nj}, 0)\rangle \quad \begin{cases} r = t,\phi \\ s = t,\phi \end{cases} \tag{11}$$

where

$$\underline{\hat{W}}^{t}_{ni} = \underline{u}_t\,(\underline{W}^{\phi}_{ni} \cdot \underline{u}_\phi) \tag{12a}$$

$$\underline{\hat{W}}^{\phi}_{ni} = \underline{u}_\phi\,(\underline{W}^{t}_{ni} \cdot \underline{u}_t) \tag{12b}$$

According to (10)-(12), Y'^{ts}_{nij} is the negative of $Y^{\phi s}_{nij}$ modified from (6) and $Y'^{\phi s}_{nij}$ is Y^{ts}_{nij} modified from (6). The expression "modified from (6)" means (6) with the t and ϕ components of $\underline{W}^{r}_{ni}$ interchanged.

Now, the matrix elements (6) are given by [2, Eq. (12)] and it is evident from [2, Eq. (9)] that

$$\underline{W}^{t}_{ni} \cdot \underline{u}_{t} = \frac{T_i(t)}{\rho} e^{-jn\phi} \tag{13a}$$

$$\underline{W}^{\phi}_{ni} \cdot \underline{u}_{\phi} = \frac{P_i(t)}{\rho_i} e^{-jn\phi} \tag{13b}$$

Hence, (10) becomes

$$Y'^{tt}_{nij} = - \hat{Y}^{\phi t}_{nij} \tag{14a}$$

$$Y'^{\phi t}_{nij} = \hat{Y}^{tt}_{nij} \tag{14b}$$

$$Y'^{t\phi}_{nij} = - \hat{Y}^{\phi\phi}_{nij} \tag{14c}$$

$$Y'^{\phi\phi}_{nij} = \hat{Y}^{t\phi}_{nij} \tag{14d}$$

where

$$\hat{Y}^{\phi t}_{nij} = Y^{\phi t}_{nij} \text{ of [2, Eq. (12b)] with } \frac{P_i(t)}{\rho_i} \text{ replaced by } \frac{T_i(t)}{\rho} \tag{15a}$$

$$\hat{Y}^{tt}_{nij} = Y^{tt}_{nij} \text{ of [2, Eq. (12a)] with } \frac{T_i(t)}{\rho} \text{ replaced by } \frac{P_i(t)}{\rho_i} \tag{15b}$$

$$\hat{Y}^{\phi\phi}_{nij} = Y^{\phi\phi}_{nij} \text{ of [2, Eq. (12d)] with } \frac{P_i(t)}{\rho_i} \text{ replaced by } \frac{T_i(t)}{\rho} \tag{15c}$$

$$\hat{Y}^{t\phi}_{nij} = Y^{t\phi}_{nij} \text{ of [2, Eq. (12c)] with } \frac{T_i(t)}{\rho} \text{ replaced by } \frac{P_i(t)}{\rho_i} \tag{15d}$$

Obviously, the interchange of functions in (15) must be accompanied by appropriate changes in the limits of integration with respect to t in [2, Eq. (12)]. Also, because the magnetic fields in [2, Eq. (12)] as it stands are evaluated on the side of the electric current sheet where the tail of $\underline{n}$ is, the Ampere's law contributions to [2, Eq. (12)] should be multiplied by the input argument IN of YZA. These contributions are the single integrals with respect to t in [2, Eq. (12)].

Consequently, the Ampere's law contributions to (14) are given by

$$\bar{Y}'^{\phi t}_{nij} = \frac{\pi(IN)}{\rho_i} \int_{t_i^-}^{t_{i+1}^-} P_i(t)\, T_j(t)dt \tag{16a}$$

$$\bar{Y}'^{t\phi}_{nij} = -\frac{\pi(IN)}{\rho_j} \int_{t_j^-}^{t_{j+1}^-} T_i(t)\, P_j(t)dt \tag{16b}$$

where the horizontal bar on the left-hand sides of (16) denotes Ampere's law contribution. If the qth interval is defined to be (t_q^-, t_{q+1}^-), then the contributions to (16) due to the integrations over the qth interval are given by

$$\dot{Y}'^{\phi t}_{nqj} = \frac{\pi(IN)}{\rho_q} \int_{t_q^-}^{t_{q+1}^-} P_q(t)\, T_j(t)dt \qquad \begin{array}{l} j = q-1,\ q \\ j \neq 0 \\ j \neq NP - 1 \end{array} \tag{17a}$$

$$\dot{Y}'^{t\phi}_{niq} = -\frac{\pi(IN)}{\rho_q} \int_{t_q^-}^{t_{q+1}^-} T_i(t)\, P_q(t)dt \qquad \begin{array}{l} i = q-1,\ q \\ i \neq 0 \\ i \neq NP - 1 \end{array} \tag{17b}$$

where NP is one of the input arguments of YZA. The dot on the left-hand sides of (17) denotes Ampere's law contribution due to integration over the qth interval. Thanks to the definitions [2, Eqs. (5) and (6)], (17) reduces to

$$\dot{Y}'^{\phi t}_{nqj} = \frac{\Delta_q \pi(IN)}{2\rho_q} \qquad \begin{cases} j = q-1,\ q \\ j \neq 0 \\ j \neq NP - 1 \end{cases} \tag{18a}$$

$$\dot{Y}'^{t\phi}_{niq} = -\frac{\Delta_q \pi(IN)}{2\rho_q} \qquad \begin{cases} i = q-1,\ q \\ i \neq 0 \\ i \neq NP - 1 \end{cases} \tag{18b}$$

where

$$\Delta_q = \bar{t}_{q+1} - \bar{t}_q \tag{19}$$

Expressions (18) are the Ampere's law contributions to the matrix elements Y'^{rs}_{nij} of (14) due to the integrations over the qth interval.

The contributions to (14) referred to in this paragraph and the next paragraph are exclusive of the Ampere's law contributions. If the pqth region is the region for which

$$\bar{t}_p \leq t \leq \bar{t}_{p+1}$$

$$\bar{t}_q \leq t' \leq \bar{t}_{q+1}$$

then the contributions to (14) due to the integrations over the pqth region are given by expressions similar to [2, Eq. (18)]. If the generating curve of the surface of revolution is assumed to be a series of straight line segments connecting the points $\bar{t}_1$, $\bar{t}_2$,... $\bar{t}_{NP}$ and if the integrations with respect to t are approximated by sampling the integrands at $t = t_p$ and multiplying by Δ_p, then the contributions to (14) due to the integrations over the pqth region reduce to

$$\overset{*,tt}{Y}_{nij} = -\frac{1}{2}\ \text{(right-hand side of [2, Eq. (22b)])} \tag{20a}$$

$$\overset{*,\phi t}{Y}_{npj} = 2\ \text{(right-hand side of [2, Eq. (22a)])} \tag{20b}$$

$$\overset{*,t\phi}{Y}_{niq} = -\frac{1}{2}\ \text{(right-hand side of [2, Eq. (22d)] without the } \delta_{pq} \text{ term)} \tag{20c}$$

$$\overset{*,\phi\phi}{Y}_{npq} = 2\ \text{(right-hand side of [2, Eq. (22c)])} \tag{20d}$$

The asterisk on the left-hand sides of (20) denotes contribution due to integration over the pqth region. The ranges of values of i and j in (20) are given by

$$\left.\begin{array}{l} i = p-1,\ p \\ i \neq 0 \\ i = NP-1 \end{array}\right\} \tag{21a}$$

$$\left.\begin{array}{l} j = q-1,\ q \\ j \neq 0 \\ j \neq NP-1 \end{array}\right\} \tag{21b}$$

The factors $\frac{1}{2}$ and 2 in (20) are due to sampling the integrands at $t = t_p$. When $t = t_p$, the functions $\frac{P_i(t)}{\rho_i}$ and $\frac{T_i(t)}{\rho}$ which are being interchanged in (15) reduce to

$$\frac{P_i(t_p)}{\rho_i} = \begin{cases} \dfrac{1}{\rho_p}, & i = p \\ 0 & i \neq p \end{cases} \tag{22}$$

$$\frac{T_i(t_p)}{\rho} = \begin{cases} \dfrac{1}{2\rho_p}, & i = p-1,\ p \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

Since [2, Eq. (22)] is equal to [2, Eq. (24)], (20) can be rewritten as

$$\overset{*}{Y}{}^{,tt}_{n,p-1,q-1} = -\frac{1}{2}\,(UC - UD)\ ,\ p \neq 0,\ q \neq 0 \tag{24a}$$

$$\overset{*}{Y}{}^{,tt}_{np,q-1} = -\frac{1}{2}\,(UC - UD)\ ,\ p \neq NP-1,\ q \neq 0 \tag{24b}$$

$$\overset{*}{Y}{}^{,tt}_{n,p-1,q} = -\frac{1}{2}\,(UC + UD)\ ,\ p \neq 0,\ q \neq NP-1 \tag{24c}$$

$$\overset{*}{Y}{}^{,tt}_{npq} = -\frac{1}{2}\,(UC + UD)\ ,\ p \neq NP-1,\ q \neq NP-1 \tag{24d}$$

$$\overset{*}{Y}{}^{,\phi t}_{np,q-1} = 2\,(UA)\ ,\ q \neq 0 \tag{24e}$$

$$\overset{*}{Y}{}^{,\phi t}_{npq} = 2\,(UB)\ ,\ q \neq NP-1 \tag{24f}$$

$$\overset{*}{Y}{}^{,t\phi}_{n,p-1,q} = -\frac{1}{2}\,(UF)\ ,\ p \neq 0 \tag{24g}$$

$$\overset{*}{Y}{}^{,t\phi}_{npq} = -\frac{1}{2}\,(UF)\ ,\ p \neq NP-1 \tag{24h}$$

$$\overset{*}{Y}{}^{,\phi\phi}_{npq} = 2\,(UE) \tag{24i}$$

where

UA = right-hand side of [2, Eq. (24a)] for $j = q-1$ (25a)

UB = right-hand side of [2, Eq. (24a)] for $j = q$ (25b)

UC−UD = right-hand side of [2, Eq. (24b)] for $j = q-1$ (25c)

UC+UD = right-hand side of [2, Eq. (24b)] for $j = q$ (25d)

UE = right-hand side of [2, Eq. (24c)] (25e)

UF = right-hand side of [2, Eq. (24d)] without the δ_{pq} term (25f)

The notation UA, UB,...UF on the left-hand sides of (25) is the same as the notation used in YZ of [2]. Expressions (24) are the contributions to the matrix elements Y'^{rs}_{nij} of (14) due to the integrations over the pqth region. No Ampere's law contributions are included in (24). The Ampere's law contributions are given by (18).

The Ampere's law contributions in (18) are different from those in [2] which consist of [2, Eq. (23)] and the δ_{pq} term in [2, Eq. (24d)]. Hence, the statements which realize the Ampere's law contributions in YZA are different from those in YZ of [2]. Lines 50-51, 74-75, 86-88, 93-99, and 424-429 of YZ of [2] obtain the Ampere's law contributions. Line 50 of YZA puts π(IN) of (18) in PN1. Line 73 puts the right-hand side of (18a) in P1.

The index JQ of DO loop 15 obtains the subscript q which appears in (18a) and (24). The index IP of DO loop 16 obtains the subscript p which appears in (24). The purpose of the statement IF(IP.NE.JQ) in lines 121, 270, and 424 of YZ of [2] is to check if the IPth interval coincides with the JQth interval. If the IPth and JQth intervals coincide, then the numerical evaluation of the integrals $G_{m\alpha}$ in [2, Section III] is affected and Ampere's law contributions are taken into account. However, YZ of [2] was not designed to accommodate a generating curve which closes upon itself.

The subroutine YZA allows for a generating curve which closes upon itself. Such a curve is treated in the following manner. A triangle function whose peak is at the first data point is needed and is obtained by overlapping the last interval of the generating curve with the first interval as in [1, Fig. 5]. This triangle function is called $T_{NP-2}(t)$. In addition to $T_{NP-2}(t)$, the process of overlapping obtains the pulse function $P_{NP-1}(t)$. $P_{NP-1}(t)$ is not wanted because it is identical to $P_1(t)$. The effect of $P_{NP-1}(t)$ can be eliminated by discarding all matrix elements (3) and (4) for which either $\underline{W}^r_{ni}$ or $\underline{J}^s_{nj}$ contain $P_{NP-1}(t)$. These elements must be discarded after exit from YZA, because YZA does not contain any logic for discarding matrix elements.

Unfortunately, (18) was derived for a generating curve which does not overlap on itself. If the last interval of the generating curve overlaps the first interval, the quantities on the right-hand side of (18) are still correct but more values of i and j are needed in (18) when $q = 1$ and $q = NP-1$ to allow for the overlapping so as to account for all Ampere's law contributions to the matrix elements (3). It will be shown that all Ampere's law contributions can be accounted for by defining ZIP to be the electrical distance from the center of the JQth interval to the center of the IPth interval and using the statement IF(ZIP.NE.0.) instead of the statement IF(IP.NE.JQ). The statement IF(ZIP.NE.0.) appears in lines 110, 259, and 408. If the generating curve does not overlap on itself, the action of the statement IF(ZIP.NE.0.) is the same as the action of the statement IF(IP.NE.JQ) and all Ampere's law contributions are accounted for. However, if the last interval of the generating curve overlaps the first interval, then it remains to be shown that (ZIP = 0.) obtains all Ampere's law contributions.

If the last interval of the generating curve overlaps the first interval, then ZIP is zero not only for

$$IP = JQ = m, \qquad m=1,2,\ldots NP-1 \qquad \textbf{Case 1}$$

but also for

$$\left.\begin{array}{l} IP = 1 \\ JQ = NP-1 \end{array}\right\} \qquad \textbf{Case 2}$$

and for

$$\left.\begin{array}{l} IP = NP-1 \\ JQ = 1 \end{array}\right\} \qquad \textbf{Case 3}$$

In what is to follow, the matrix elements Y'^{rs}_{nij} of (3) are viewed as interactions between testing functions and expansion functions. The combination of Cases 1, 2, and 3 will give all Ampere's law contributions if it covers all possible interactions between parts of testing functions on the mth interval and parts of expansion functions on the mth interval for $m = 1,2,\ldots NP-2$.

With regard to the Ampere's law contributions (18), the last statement is more general than necessary because not all interactions between testing functions and expansion functions are involved in (18). For example, the interactions Y'^{tt}_{nij} between t directed testing functions and t directed expansion functions are not involved in (18) and neither are the interactions $Y'^{\phi\phi}_{nij}$ between ϕ directed testing functions and ϕ directed expansion functions. However, the matrix elements Y^{tt}_{nij} and $Y^{\phi\phi}_{nij}$ of (6) calculated by YZ of [2] do have Ampere's law contributions. It would be useful to know that YZ of [2] can be modified to allow for a closed generating curve by using the same technique as in YZA. This technique consists of overlapping the last interval of the generating curve with the first, replacing the statement IF(IP.NE.JQ) by the statement IF(ZIP.NE.O.), and deleting the matrix elements associated with $P_{NP-1}(t)$ after exit from the subroutine.

It is evident that the portion of Case 1 for which

$$IP = JQ = m, \; m = 2,\ldots NP-2$$

covers all possible interactions between parts of testing functions on the mth interval and expansion functions on the mth interval for $m = 2,3,\ldots NP-2$. However, clarification is needed with regard to the first interval because, due to the overlapping, the first interval is sometimes disguised as the (NP-1)th interval.

All possible interactions between parts of testing functions on the first interval and parts of expansion functions on the first interval are listed as

1) $T^b_{NP-2}(t)$ with $T^b_{NP-2}(t)$
2) $T^a_1(t)$ with $T^b_{NP-2}(t)$
3) $P_1(t)$ with $T^b_{NP-2}(t)$
4) $T^b_{NP-2}(t)$ with $T^a_1(t)$
5) $T^a_1(t)$ with $T^a_1(t)$
6) $P_1(t)$ with $T^a_1(t)$
7) $T^b_{NP-2}(t)$ with $P_1(t)$
8) $T^a_1(t)$ with $P_1(t)$
9) $P_1(t)$ with $P_1(t)$

In each of the foregoing 9 interactions, the first function is the part of the testing function and the second function is the part of the expansion function. $T^b_{NP-2}(t)$ is the second part of the (NP-2)th triangle function, the downward sloping part. $T_1^a(t)$ is the first part of the first triangle function, the upward sloping part.

The interactions covered by Cases 2 and 3 and the parts of Case 1 for which IP = JQ = 1 and IP = JQ = NP-1 are listed as

$$\left.\begin{array}{rlll} 2) & T_1^a(t) & \text{with} & T_{NP-2}^b(t) \\ 3) & P_1(t) & \text{with} & T_{NP-2}^b(t) \\ 10) & T_1^a(t) & \text{with} & P_{NP-1}(t) \\ 11) & P_1(t) & \text{with} & P_{NP-1}(t) \end{array}\right\} \begin{array}{l} IP = 1 \\ JQ = NP-1 \end{array} \qquad \text{Case 2}$$

$$\left.\begin{array}{rlll} 4) & T_{NP-2}^b(t) & \text{with} & T_1^a(t) \\ 12) & P_{NP-1}(t) & \text{with} & T_1^a(t) \\ 7) & T_{NP-2}^b(t) & \text{with} & P_1(t) \\ 13) & P_{NP-1}(t) & \text{with} & P_1(t) \end{array}\right\} \begin{array}{l} IP = NP-1 \\ JQ = 1 \end{array} \qquad \text{Case 3}$$

$$\left.\begin{array}{rlll} 5) & T_1^a(t) & \text{with} & T_1^a(t) \\ 6) & P_1(t) & \text{with} & T_1^a(t) \\ 8) & T_1^a(t) & \text{with} & P_1(t) \\ 9) & P_1(t) & \text{with} & P_1(t) \end{array}\right\} IP = JQ = 1 \qquad \text{Part of Case 1}$$

$$\left.\begin{array}{rlll} 1) & T_{NP-2}^b(t) & \text{with} & T_{NP-2}^b(t) \\ 14) & P_{NP-1}(t) & \text{with} & T_{NP-2}^b(t) \\ 15) & T_{NP-2}^b(t) & \text{with} & P_{NP-1}(t) \\ 16) & P_{NP-1}(t) & \text{with} & P_{NP-1}(t) \end{array}\right\} IP = JQ = NP-1 \qquad \text{Part of Case 1}$$

The preceding 16 interactions were numbered so as to facilitate comparison with the 9 interactions in the last paragraph.

Of the 16 interactions in the last paragraph, numbers 1 to 9 are the interactions in the second from the last paragraph and numbers 10 to 16 are to be discarded because they contain $P_{NP-1}(t)$. Drawing an equivalence between the interactions in the last paragraph and the interactions in the second from the last paragraph, we conclude that the combination of Cases 1, 2. and 3 covers all possible interactions between parts of testing functions on the mth interval and parts of expansion functions on the mth interval for m = 1,2,... NP-2. Hence, (ZIP = 0.) obtains all Ampere's law contributions when the last interval on the generating curve overlaps the first interval. If such overlapping is intended, then the input arguments RH and ZH of YZA must satisfy

$$\left.\begin{aligned} RH(NP-1) &= RH(1) \\ RH(NP) &= RH(2) \\ ZH(NP-1) &= ZH(1) \\ ZH(NP) &= ZH(2) \end{aligned}\right\} \qquad (26)$$

exactly. Otherwise, the computed values of ZIP will not be zero in Cases 2 and 3.

With the intention of showing that YZA implements (24), we list in Table 1 some variables in YZA whose values are different from those in YZ of [2].

Table 1. Comparison of some variables in YZA with those in YZ of [2].

Line in YZA	Variable in YZA	Expression in terms of variables in YZ of [2]
379	W1	2.*W1
380	W2	2.*W2
385	H1C	-.5*H1C
387	H3C	2.*H3C
388	H2C	2.*H2C
389	H3C	2.*H3C
390	W3	-.5*W3
391	W4	-.5*W4
392	W5	-.5*W5
405	UC	2.*UC
406	UB	2.*UB
407	UF	-.5*UF
411	UA	2.*UA
412	UB	2.*UB
415	UC	-.5*(UC-UD)
416	UD	-.5*(UC+UD)
417	UE	2.*UE

In Table 1, the variable in the second column is defined by the statement in YZA whose line number is given in the first column. The expression in the third column is the value of the variable in the second column. The values of the variables appearing in the third column are the values which exist in YZ just after the counterpart to the variable in the second column has been defined in YZ. For example, the value of W1 in the first line of the third column is defined in line 391 of YZ of [2]. As an exception, the value of UD intended in the third from the last line of the third column is that defined by line 433 of YZ of [2]. Table 1 was constructed under the assumption that no Ampere's law contributions come into play. This amounts to assuming that lines 425-429 of YZ of [2] are not executed and that lines 409-410 of YZA are not executed. The Ampere's law contributions (18) will be considered later.

The last 6 variables in the second column of Table 1 are used to construct the matrix elements Y'^{rs}_{nij} of (3). The statements in YZA which perform this task are listed in Table 2.

Table 2. Construction of matrix elements Y'^{rs}_{nij}

Line in YZA	Statement in YZA	Matrix element
461,476	Y(K1) = Y(K1) + UC	$Y'^{tt}_{n,p-1,q-1}$
466,483	Y(K2) = Y(K2) + UC	$Y'^{tt}_{n,p,q-1}$
448,477	Y(K3) = Y(K3) + UD	$Y'^{tt}_{n,p-1,q}$
453,484	Y(K4) = UD	Y'^{tt}_{npq}
458,471	Y(K5) = Y(K5) + UA	$Y'^{\phi t}_{np,q-1}$
445,472	Y(K6) = UB	$Y'^{\phi t}_{npq}$
449,462,478	Y(K7) = Y(K7) + UF	$Y'^{t\phi}_{n,p-1,q}$
454,467,485	Y(K8) = UF	$Y'^{t\phi}_{npq}$
489	Y(K9) = UE	$Y'^{\phi\phi}_{npq}$

In Table 2, the variable being modified by the statement in the second column represents the matrix element in the third column. For example, Y(K1) which appears in the first line of the second column of Table 2 is the storage location of the matrix element $Y'^{tt}_{n,p-1,q-1}$. The subscripts n,p, and q appearing in the third column of Table 2 are given in terms of variables in YZA by

$$n = M + M1-1$$

$$p = IP$$

$$q = JQ$$

Here, M is the index of DO loop 31, M1 is one of the input arguments of YZA, IP is the index of DO loop 16, and JQ is the index of DO loop 15. In view of the fact that the variables UA, UB, UC, UD, UE, and UF appearing in the second column of Table 2 are given by the last 6 entries of the third column of Table 1, it is now evident that YZA implements (24).

From (18), the correct values of the Ampere's law contributions are $\pm(\Delta_q \pi(IN))/(2\rho_q)$. In the paragraph which follows (25) we established that line 73 stores $(\Delta_q \pi(IN))/(2\rho_q)$ in P1. In the paragraph prior to the introduction of Table 1, we concluded that the Ampere's law contributions come into play whenever ZIP = 0. Accordingly, lines 409-410 are executed when ZIP = 0. The effect of line 409 is to add P1 to the values of UA and UB calculated by lines 411 and 412. Line 410 subtracts P1 from the value of UF calculated by line 407. The previously mentioned variables UA, UB, and UF appear in the second column of Table 1 and were transferred to the second column of Table 2. It is now evident from Table 2 that execution of lines 409-410 adds P1 to $Y'^{\phi t}_{np,q-1}$ and $Y'^{\phi t}_{npq}$ and subtracts P1 from $Y'^{t\phi}_{n,p-1,q}$ and $Y'^{t\phi}_{npq}$. Hence, YZ takes proper account of the Ampere's law contributions.

```
C     LISTING OF THE SUBROUTINE YZA
C     THE SUBROUTINE YZA CALLS THE FUNCTION BLOG
      SUBROUTINE YZA(M1,M2,NP,NPHI,NT,IN,RH,ZH,X,A,XT,AT,Y,Z)
      COMPLEX Y(2209),Z(2209),U,U1,U2,U3,U4,U5,U6,H1A,H2A,H3A,GA(48)
      COMPLEX GB(48),GC(48),GD(48),GE(48),H1B,H2B,H3B,H1C,H2C,H3C,H4A
      COMPLEX H5A,H6A,H4B,H5B,H6B,UA,UB,UC,UD,UE,UF,G1A(10),G2A(10)
      COMPLEX G3A(10),G1B(10),G2B(10),G3B(10),G1C(10),G2C(10),G3C(10)
      COMPLEX G4A(10),G5A(10),G6A(10),G4B(10),G5B(10),G6B(10),CMPLX
      DIMENSION RH(43),ZH(43),X(48),A(48),XT(10),AT(10),RS(42),ZS(42)
      DIMENSION D(42),DR(42),DZ(42),DM(42),C1(48),C2(48),C3(48),C4(200)
      DIMENSION C5(200),C6(200),R2(10),Z2(10),R7(10),Z7(10)
      CT=2.
      CP=.1
      DO 10 I=2,NP
      I2=I-1
      RS(I2)=.5*(RH(I)+RH(I2))
      IF(RS(I2).EQ.0.) RS(I2)=1.
      ZS(I2)=.5*(ZH(I)+ZH(I2))
      D1=.5*(RH(I)-RH(I2))
      D2=.5*(ZH(I)-ZH(I2))
      D(I2)=SQRT(D1*D1+D2*D2)
      DR(I2)=D1
      DZ(I2)=D2
      DM(I2)=D(I2)/RS(I2)
   10 CONTINUE
      M3=M2-M1+1
      M4=M1-1
      PI2=1.570796
      PP=9.869604
      DO 11 K=1,NPHI
      PH=PI2*(X(K)+1.)
      C1(K)=PH
      C2(K)=PH*PH
      SN=SIN(.5*PH)
      C3(K)=4.*SN*SN
      A1=PI2*A(K)
      D4=.5*A1*C3(K)
      D5=A1*COS(PH)
      D6=A1*SIN(PH)
      M5=K
      DO 29 M=1,M3
      PHM=(M4+M)*PH
      A2=COS(PHM)
      C4(M5)=D4*A2
      C5(M5)=D5*A2
      C6(M5)=D6*SIN(PHM)
      M5=M5+NPHI
   29 CONTINUE
   11 CONTINUE
      PN1=3.141593*IN
      U=(0.,1.)
      U1=.5*U
      U2=2.*U
      MP=NP-1
      MT=NP-1
      N=MT+MP
      N2N=MT*N
      N2=N*N
      JN=-1-N
      DO 15 JO=1,MP
```

```
      KQ=2
      IF(JQ.EQ.1) KQ=1
      IF(JQ.EQ.MP) KQ=3
      R1=RS(JQ)
      Z1=ZS(JQ)
      D1=D(JQ)
      D2=DR(JQ)
      D3=DZ(JQ)
      D4=D2/R1
      D5=D1/R1
      SV=D2/D1
      CV=D3/D1
      P1=PN1*D5
      P3=2.*D1
      P4=2.*D4
      P5=D4*D4
      P6=D1*D1
      P7=P6*D1
      T6=CT*D1
      T62=T6+D1
      T62=T62*T62
      R6=CP*R1
      R62=R6*R6
      DO 12 L=1,NT
      D6=XT(L)
      R2(L)=R1+D2*D6
      Z2(L)=Z1+D3*D6
   12 CONTINUE
      DO 16 IP=1,MP
      R3=RS(IP)
      Z3=ZS(IP)
      R4=R1-R3
      Z4=Z1-Z3
      U3=D2*U1
      U4=D3*U1
      DO 40 L=1,NT
      D7=R2(L)-R3
      D8=Z2(L)-Z3
      R7(L)=R3*R2(L)
      Z7(L)=D7*D7+D8*D8
   40 CONTINUE
      PH=R4*SV+Z4*CV
      A1=ABS(PH)
      A2=ABS(R4*CV-Z4*SV)
      D6=A2
      IF(A1.LE.D1) GO TO 26
      D6=A1-D1
      D6=SQRT(D6*D6+A2*A2)
   26 ZIP=R4*R4+Z4*Z4
      IF(ZIP.NE.0..AND.(R6.GT.D6.OR.T6.LE.D6)) GO TO 41
      Z5=ZIP
      R5=R3*R1
      PHM=.5*R3*SV
      DO 33 K=1,NPHI
      A1=C3(K)
      RR=Z5+R5*A1
      H1A=0.
      H2A=0.
      H3A=0.
      H4A=0.
```

```
      H5A=0.
      IF(RR.LT.T62) GO TO 34
      DO 35 L=1,NT
      W=Z7(L)+R7(L)*A1
      R=SQRT(W)
      SN=-SIN(R)
      CS=COS(R)
      D6=AT(L)/R
      H1B=D6/W*CMPLX(CS-R*SN,SN+R*CS)
      H1A=H1B+H1A
      H2B=XT(L)*H1B
      H2A=H2B+H2A
      H3A=XT(L)*H2B+H3A
      H4B=D6*CMPLX(CS,SN)
      H4A=H4B+H4A
      H5A=XT(L)*H4B+H5A
   35 CONTINUE
      GO TO 36
   34 DO 37 L=1,NT
      W=Z7(L)+R7(L)*A1
      R=SQRT(W)
      IF(R.GT..5) GO TO 14
      CS=R*(W*(.6944444E-2-W*.1736111E-3)-.125)
      SN=W*(.3333333E-1-W*.1190476E-2)-.3333333
      H1B=AT(L)*CMPLX(CS,SN)
      CS=R*(W*(.4166667E-1-.1388889E-2*W)-.5)
      SN=W*(W*(.1984126E-3*W-.8333333E-2)+.1666667)-1.
      H4B=AT(L)*CMPLX(CS,SN)
      GO TO 43
   14 SN=-SIN(R)
      CS=COS(R)
      D6=AT(L)/R
      H1B=D6*((CMPLX(CS-R*SN,SN+R*CS)-1.)/W-.5)
      H4B=D6*CMPLX(CS-1.,SN)
   43 H1A=H1B+H1A
      H2B=XT(L)*H1B
      H2A=H2B+H2A
      H3A=XT(L)*H2B+H3A
      H4A=H4B+H4A
      H5A=XT(L)*H4B+H5A
   37 CONTINUE
      A1=PH+PHM*A1
      A2=ABS(A1)
      R=RR-A2*A2
      D6=A2-D1
      D7=A2+D1
      D62=D6*D6
      D72=D7*D7
      D8=SQRT(D62+R)
      D9=SQRT(D72+R)
      IF(R-(RR*1.E-5)) 52,52,53
   52 IF(D6.LT.0.) STOP
      W4=.5/D62-.5/D72
      GO TO 54
   53 W4=(D7/D9-D6/D8)/R
   54 IF(D6.GE.0.) GO TO 38
      W=ALOG((D7+D9)*(-D6+D8)/R)
      GO TO 39
   38 W=ALOG((D7+D9)/(D6+D8))
   39 W1=(W4+.5*W)/D1
```

```
      W5=A2/D1
      W2=(.5*(D9-D8)-1./D9+1./D8)/P6-W5*W1
      W3=(.25*(D7*D9-D6*D8)+W-R*(W4+.25*W))/P7-W5*(2.*W2+W5*W1)
      W4=W/D1
      W5=(D9-D8-A2*W)/P6
      IF(A1.GE.0.) GO TO 27
      W2=-W2
      W5=-W5
   27 H1A=W1+H1A
      H2A=W2+H2A
      H3A=W3+H3A
      H4A=W4+H4A
      H5A=W5+H5A
   36 GA(K)=H1A
      GB(K)=H2A
      GC(K)=H3A
      GD(K)=H4A
      GE(K)=H5A
   33 CONTINUE
      K1=0
      DO 45 N=1,M3
      H1A=0.
      H2A=0.
      H3A=0.
      H1B=0.
      H2B=0.
      H3B=0.
      H1C=0.
      H2C=0.
      H3C=0.
      H4A=0.
      H5A=0.
      H6A=0.
      H4B=0.
      H5B=0.
      H6B=0.
      DO 46 K=1,NPHI
      K1=K1+1
      D6=C4(K1)
      D7=C5(K1)
      D8=C6(K1)
      UA=GA(K)
      UB=GB(K)
      UC=GC(K)
      UD=GD(K)
      UE=GE(K)
      H1A=D6*UA+H1A
      H2A=D7*UA+H2A
      H3A=D8*UA+H3A
      H1B=D6*UB+H1B
      H2B=D7*UB+H2B
      H3B=D8*UB+H3B
      H1C=D6*UC+H1C
      H2C=D7*UC+H2C
      H3C=D8*UC+H3C
      H4A=D6*UD+H4A
      H5A=D7*UD+H5A
      H6A=D8*UD+H6A
      H4B=D6*UE+H4B
      H5B=D7*UE+H5B
```

```
      H6B=D8*UE+H6B
   46 CONTINUE
      G1A(M)=H1A
      G2A(M)=H2A
      G3A(M)=H3A
      G1B(M)=H1B
      G2B(M)=H2B
      G3B(M)=H3B
      G1C(M)=H1C
      G2C(M)=H2C
      G3C(M)=H3C
      G4A(M)=H4A
      G5A(M)=H5A
      G6A(M)=H6A
      G4B(M)=H4B
      G5B(M)=H5B
      G6B(M)=H6B
   45 CONTINUE
      IF(ZIP.NE.0.) GO TO 47
      A1=D5*D5
      D8=0.
      D9=0.
      DO 63 K=1,NPHI
      D8=D8+A(K)/SQRT(C2(K)+A1)
      D9=D9+A(K)*BLOG(D5/C1(K))
   63 CONTINUE
      A2=3.141593/D5
      D8=(BLOG(A2)-PI2*D8)/(R5*R1)
      D9=2./R1*(BLOG(A2)+A2*BLOG(1./A2))-3.141593/D1*D9
      DO 67 M=1,M3
      G1A(M)=D8+G1A(M)
      G2A(M)=0.
      G2B(M)=0.
      G2C(M)=0.
      G3A(M)=0.
      G5A(M)=D9+G5A(M)
   67 CONTINUE
      GO TO 47
   41 DO 25 M=1,M3
      G1A(M)=0.
      G2A(M)=0.
      G3A(M)=0.
      G1B(M)=0.
      G2B(M)=0.
      G3B(M)=0.
      G1C(M)=0.
      G2C(M)=0.
      G3C(M)=0.
      G4A(M)=0.
      G5A(M)=0.
      G6A(M)=0.
      G4B(M)=0.
      G5B(M)=0.
      G6B(M)=0.
   25 CONTINUE
      DO 13 L=1,NT
      R5=R7(L)
      Z5=Z7(L)
      DO 17 K=1,NPHI
      W=Z5+R5*C3(K)
```

```
      R=SQRT(W)
      SN=-SIN(R)
      CS=COS(R)
      GA(K)=CMPLX(CS-R*SN,SN+R*CS)/(W*R)
      GD(K)=CMPLX(CS,SN)/R
   17 CONTINUE
      IF(R62.LE.Z5) GO TO 51
      D6=0.
      D7=0.
      D9=0.
      DO 62 K=1,NPHI
      W2=C2(K)
      W=1./(Z5+R5*W2)
      W1=A(K)*SQRT(W)
      D6=D6+W1*W2*W
      D7=D7+W1*(.5+W*(1.+.125*W*R5*W2*W2))
      D9=D9+W1
   62 CONTINUE
      W1=R5/Z5
      W2=PP*W1
      W=SQRT(W2)
      W3=1.+W2
      R=SQRT(W3)
      W4=SQRT(R5)
      W5=ALOG(W+R)
      D8=-PI2*D6-(W/R-W5)/(R5*W4)
      D6=.5*D8
      D7=((W/R*(W1-(.125+.1666667*W2)/W3)+.125*W5)/R5+.5*W5)/W4-PI2*D7
      D9=W5/W4-PI2*D9
   51 A1=AT(L)
      A2=XT(L)*A1
      A3=XT(L)*A2
      K1=0
      DO 30 M=1,M3
      W=M+M4
      H1A=0.
      H2A=0.
      H3A=0.
      H4A=0.
      H5A=0.
      H6A=0.
      DO 32 K=1,NPHI
      K1=K1+1
      H1B=GA(K)
      W4=C4(K1)
      W5=C5(K1)
      W6=C6(K1)
      H1A=W4*H1B+H1A
      H2A=W5*H1B+H2A
      H3A=W6*H1B+H3A
      H1B=GD(K)
      H4A=W4*H1B+H4A
      H5A=W5*H1B+H5A
      H6A=W6*H1B+H6A
   32 CONTINUE
      IF(R62.LE.Z5) GO TO 44
      H1A=D6+H1A
      H2A=D7-(W*W+1.)*D6+H2A
      H3A=W*D8+H3A
      H5A=D9+H5A
```

```
   44 G1A(M)=A1*H1A+G1A(M)
      G2A(M)=A1*H2A+G2A(M)
      G3A(M)=A1*H3A+G3A(M)
      G1B(M)=A2*H1A+G1B(M)
      G2B(M)=A2*H2A+G2B(M)
      G3B(M)=A2*H3A+G3B(M)
      G1C(M)=A3*H1A+G1C(M)
      G2C(M)=A3*H2A+G2C(M)
      G3C(M)=A3*H3A+G3C(M)
      G4A(M)=A1*H4A+G4A(M)
      G5A(M)=A1*H5A+G5A(M)
      G6A(M)=A1*H6A+G6A(M)
      G4B(M)=A2*H4A+G4B(M)
      G5B(M)=A2*H5A+G5B(M)
      G6B(M)=A2*H6A+G6B(M)
   30 CONTINUE
   13 CONTINUE
   47 A2=D(IP)
      W1=A2*(R4*D3-Z4*D2)
      W2=-A2*R3*D3
      A3=DZ(IP)
      D6=DR(IP)
      D7=Z4*D6
      D9=D3*D6
      H1C=(D2*(R3*A3+D7)-R1*D9)*U1
      D8=A2*D1
      H3C=D8*U2
      H2C=Z4*H3C
      H3C=D3*H3C
      W3=D1*(D7-R4*A3)
      W4=D1*(D9-D2*A3)
      W5=-D1*R1*A3
      A1=DR(IP)
      U5=A1*U3
      U6=A3*U4
      D6=-D2*A2
      D7=D1*A1
      A3=DM(IP)
      JM=JN
      DO 31 M=1,M3
      H2A=G2A(M)
      H1A=G1A(M)
      H2B=G2B(M)
      H1B=G1B(M)
      UC=W1*H2A+W2*H1A
      UB=W1*H2B+W2*H1B
      UF=W3*(H2A+D4*H2B)+W4*(H2B+D4*G2C(M))+W5*(H1A+P4*H1B+P5*G1C(M))
      IF(ZIP.NE.0.) GO TO 48
      UC=UC+P1
      UF=UF-P1
   48 UA=UC-UB
      UB=UC+UB
      H3A=G3A(M)
      H3B=G3B(M)
      UC=H1C*(H3A-H3B)
      UD=H1C*(H3A+H3B)
      UE=H2C*(H3A+D4*H3B)+H3C*(H3B+D4*G3C(M))
      H5A=G5A(M)
      H5B=G5B(M)
      H4A=G4A(M)+H5A
```

```
      H4B=G4B(M)+H5B
      H6A=G6A(M)
      H6B=G6B(M)
      H3A=U5*H5A+U6*H4A
      H1B=U5*H5B+U6*H4B
      H1A=H3A-H1B
      H2A=H3A+H1B
      H3A=-U1*H4A
      H1B=D6*H6A
      W=M+N4
      A1=W*A3
      H2B=D6*H6B-A1*H4A
      H3B=D7*(H6A+D4*H6B)
      H4A=W*D5*H4A
      K1=IP+JM
      K2=K1+1
      K3=K1+N
      K4=K2+N
      K5=K2+MT
      K6=K4+MT
      K7=K3+N2N
      K8=K4+N2N
      K9=K8+MT
      GO TO (18,20,19),KQ
   18 Y(K6)=UB
      Z(K6)=H1B+H2B
      IF(IP.EQ.1) GO TO 21
      Y(K3)=Y(K3)+UD
      Y(K7)=Y(K7)+UF
      Z(K3)=Z(K3)+H2A-H3A
      Z(K7)=Z(K7)+H3B-H4A
      IF(IP.EQ.MP) GO TO 22
   21 Y(K4)=UD
      Y(K8)=UF
      Z(K4)=H2A+H3A
      Z(K8)=H3B+H4A
      GO TO 22
   19 Y(K5)=Y(K5)+UA
      Z(K5)=Z(K5)+H1B-H2B
      IF(IP.EQ.1) GO TO 23
      Y(K1)=Y(K1)+UC
      Y(K7)=Y(K7)+UF
      Z(K1)=Z(K1)+H1A+H3A
      Z(K7)=Z(K7)+H3B-H4A
      IF(IP.EQ.MP) GO TO 22
   23 Y(K2)=Y(K2)+UC
      Y(K8)=UF
      Z(K2)=Z(K2)+H1A-H3A
      Z(K8)=H3B+H4A
      GO TO 22
   20 Y(K5)=Y(K5)+UA
      Y(K6)=UB
      Z(K5)=Z(K5)+H1B-H2B
      Z(K6)=H1B+H2B
      IF(IP.EQ.1) GO TO 24
      Y(K1)=Y(K1)+UC
      Y(K3)=Y(K3)+UD
      Y(K7)=Y(K7)+UF
      Z(K1)=Z(K1)+H1A+H3A
      Z(K3)=Z(K3)+H2A-H3A
      Z(K7)=Z(K7)+H3B-H4A
      IF(IP.EQ.MP) GO TO 22
   24 Y(K2)=Y(K2)+UC
      Y(K4)=UD
      Y(K8)=UF
      Z(K2)=Z(K2)+H1A-H3A
      Z(K4)=H2A+H3A
      Z(K8)=H3B+H4A
   22 Y(K9)=UE
      Z(K9)=U2*(D8*(H5A+D4*H5B)-A1*H4A)
      JM=JM+N2
   31 CONTINUE
   16 CONTINUE
      JN=JN+N
   15 CONTINUE
      RETURN
      END
```

III. THE FUNCTION BLOG AND THE SUBROUTINES PLANE, DECOMP, AND SOLVE

The function BLOG is exactly the same as in [3, page 56]. The only difference between the subroutine PLANE (of the present report) and the subroutine PLANE of [3, pages 57-62] is that the statement

```
IF(R1.EQ.0.)   R1 = .5
```

in line 39 of PLANE has no counterpart in the subroutine PLANE of [3, pages 57-62]. The action of this statement is similar to that of the statement in line 17 of the subroutine YZA. Except for a difference in the actual space allocated to the variables UL, SCL, and IPS, the subroutine DECOMP is the same as the subroutine DECOMP of [3, pages 63-64]. Minimum allocations in DECOMP are the same as those in the subroutine DECOMP of [3, pages 63-64]. The subroutine SOLVE differs from the subroutine SOLVE of [3, pages 63-64] only in the actual space allocated to the variables UL, B, X, and IPS. Minimum allocations in SOLVE are the same as those in the subroutine SOLVE of [3, pages 63-64].

```
C     LISTING OF THE FUNCTION BLOG
      FUNCTION BLOG(X)
      IF(X.GT..1) GO TO 1
      X2=X*X
      BLOG=((.075*X2-.1666667)*X2+1.)*X
      RETURN
    1 BLOG=ALOG(X+SQRT(1.+X*X))
      RETURN
      END
      LISTING OF THE SUBROUTINE PLANE
      SUBROUTINE PLANE(M1,M2,NF,NP,NT,RH,ZH,XT,AT,THR,R)
      COMPLEX R(240),U,U1,UA,UB,FA(10),FB(10),F2A,F2B,F1A,F1B,U2,U3,U4
      COMPLEX US,CMPLX
      DIMENSION RH(43),ZH(43),XT(10),AT(10),THR(3),CS(3),SN(3),R2(10)
      DIMENSION Z2(10),BJ(50)
      MP=NP-1
      MT=MP-1
      N=MT+MP
      N2=2*N
      DO 11 K=1,NF
      X=THR(K)
      CS(K)=COS(X)
      SN(K)=SIN(X)
   11 CONTINUE
      U=(0.,1.)
      U1=3.141593*U**M1
      M3=M1+1
      M4=M2+3
      IF(M1.EQ.0) M3=2
      M5=M1+2
      M6=M2+2
      DO 12 IP=1,MP
      K2=IP
      I=IP+1
      DR=.5*(RH(I)-RH(IP))
      DZ=.5*(ZH(I)-ZH(IP))
      D1=SQRT(DR*DR+DZ*DZ)
      R1=.25*(RH(I)+RH(IP))
      IF(R1.EQ.0.) R1=.5
      Z1=.5*(ZH(I)+ZH(IP))
      DR=.5*DR
      D2=DR/R1
      DO 13 L=1,NT
      R2(L)=R1+DR*XT(L)
      Z2(L)=Z1+DZ*XT(L)
   13 CONTINUE
      DO 14 K=1,NF
      CC=CS(K)
      SS=SN(K)
      D3=DR*CC
      D4=-DZ*SS
      D5=D1*CC
      DO 23 M=M3,M4
      FA(M)=0.
      FB(M)=0.
   23 CONTINUE
      DO 15 L=1,NT
      X=SS*R2(L)
      IF(X.GT..5E-7) GO TO 19
      DO 20 M=M3,M4
```

```
      BJ(M)=0.
   20 CONTINUE
      BJ(2)=1.
      S=1.
      GO TO 18
   19 M=2.8*X+14.-2./X
      IF(X.LT..5) M=11.8+ALOG10(X)
      IF(M.LT.M4) M=M4
      BJ(M)=0.
      JM=M-1
      BJ(JM)=1.
      DO 16 J=4,M
      J2=JM
      JM=JM-1
      J1=JM-1
      BJ(JM)=J1/X*BJ(J2)-BJ(JM+2)
   16 CONTINUE
      S=0.
      IF(M.LE.4) GO TO 24
      DO 17 J=4,M,2
      S=S+BJ(J)
   17 CONTINUE
   24 S=BJ(2)+2.*S
   18 ARG=Z2(L)*CC
      UA=AT(L)/S*CMPLX(COS(ARG),SIN(ARG))
      UB=XT(L)*UA
      DO 25 M=M3,M4
      FA(M)=BJ(M)*UA+FA(M)
      FB(M)=BJ(M)*UB+FB(M)
   25 CONTINUE
   15 CONTINUE
      IF(M1.NE.0) GO TO 26
      FA(1)=-FA(3)
      FB(1)=-FB(3)
   26 UA=U1
      DO 27 M=M5,M6
      M7=M-1
      M8=M+1
      F2A=UA*(FA(M8)+FA(M7))
      F2B=UA*(FB(M8)+FB(M7))
      UB=U*UA
      F1A=UB*(FA(M8)-FA(M7))
      F1B=UB*(FB(M8)-FB(M7))
      U4=D4*UA
      U2=D3*F1A+U4*FA(M)
      U3=D3*F1B+U4*FB(M)
      U4=DR*F2A
      U5=DR*F2B
      K1=K2-1
      K4=K1+N
      K5=K2+N
      R(K2+MT)=-D5*(F2A+D2*F2B)
      R(K5+MT)=D1*(F1A+D2*F1B)
      IF(IP.EQ.1) GO TO 21
      R(K1)=R(K1)+U2-U3
      R(K4)=R(K4)+U4-U5
      IF(IP.EQ.MP) GO TO 22
   21 R(K2)=U2+U3
      R(K5)=U4+U5
   22 K2=K2+N2
```

```
      UA=UB
   27 CONTINUE
   14 CONTINUE
   12 CONTINUE
      RETURN
      END
C     LISTING OF THE SUBROUTINE DECOMP
      SUBROUTINE DECOMP(N,IPS,UL)
      COMPLEX UL(6241),PIVOT,EM
      DIMENSION SCL(79),IPS(79)
      DO 5 I=1,N
      IPS(I)=I
      RN=0.
      J1=I
      DO 2 J=1,N
      ULM=ABS(REAL(UL(J1)))+ABS(AIMAG(UL(J1)))
      J1=J1+N
      IF(RN-ULM) 1,2,2
    1 RN=ULM
    2 CONTINUE
      SCL(I)=1./RN
    5 CONTINUE
      NM1=N-1
      K2=0
      DO 17 K=1,NM1
      BIG=0.
      DO 11 I=K,N
      IP=IPS(I)
      IPK=IP+K2
      SIZE=(ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))))*SCL(IP)
      IF(SIZE-BIG) 11,11,10
   10 BIG=SIZE
      IPV=I
   11 CONTINUE
      IF(IPV-K) 14,15,14
   14 J=IPS(K)
      IPS(K)=IPS(IPV)
      IPS(IPV)=J
   15 KPP=IPS(K)+K2
      PIVOT=UL(KPP)
      KP1=K+1
      DO 16 I=KP1,N
      KP=KPP
      IP=IPS(I)+K2
      EM=-UL(IP)/PIVOT
   18 UL(IP)=-EM
      DO 16 J=KP1,N
      IP=IP+N
      KP=KP+N
      UL(IP)=UL(IP)+EM*UL(KP)
   16 CONTINUE
      K2=K2+N
   17 CONTINUE
      RETURN
      END
      LISTING OF THE SUBROUTINE SOLVE
      SUBROUTINE SOLVE(N,IPS,UL,B,X)
      COMPLEX UL(6241),B(79),X(79),SUM
      DIMENSION IPS(79)
      NP1=N+1
      IP=IPS(1)
      X(1)=B(IP)
      DO 2 I=2,N
      IP=IPS(I)
      IPB=IP
      IM1=I-1
      SUM=0.
      DO 1 J=1,IM1
      SUM=SUM+UL(IP)*X(J)
    1 IP=IP+N
    2 X(I)=B(IPB)-SUM
      K2=N*(N-1)
      IP=IPS(N)+K2
      X(N)=X(N)/UL(IP)
      DO 4 IBACK=2,N
      I=NP1-IBACK
      K2=K2-N
      IPI=IPS(I)+K2
      IP1=I+1
      SUM=0.
      IP=IPI
      DO 3 J=IP1,N
      IP=IP+N
    3 SUM=SUM+UL(IP)*X(J)
    4 X(I)=(X(I)-SUM)/UL(IPI)
      RETURN
      END
```

IV. THE SUBROUTINE PRNT

The subroutine PRNT uses knowledge of the superscripted I_{nj}'s or V_{nj}'s and the $k^+ \bar{\rho}^{\pm}$'s in a portion of [1, Eqs. (87a), (88a), and (89a)] and in the corresponding portion of [1, Eqs. (87b), (88b), and (89b)] in order to calculate and print out these portions of [1, Eqs. (87)-(89)]. By calling the subroutine PRNT repeatedly, it is possible to print out all the quantities on the right-hand sides of [1, Eqs. (87)-(89)] except those for the cases in which $j = 1$ and $j = N^+$ in [1, Eq. (87a)] and $j = M^+ + 1$ and $j = M^+ + M$ in [1, Eq. (89a)]. PRNT is not designed to treat these cases. They are included in [1, Eqs. (87a) and (89a)] merely for convenience.

Except for the cases in which $j = 1$ and $j = N^+$ in [1, Eq. (87a)] and $j = M^+ + 1$ and $j = M^+ + M$ in [1, Eq. (89a)], any portion of [1, Eqs. (87a), (88a), and (89a)] can be written as

C1 * XX(J + JX)/RA(J + J3), J = 1,2,...J1 (27)

and the corresponding portion of [1, Eqs. (87b), (88b), and (89b)] can be written as

C2 * XX (J+J1+JX)/(RA(J+J4-1) + RA(J+J4)), J = 1,2,...J2 (28)

In (27) and (28), XX represents the I_{nj}'s or V_{nj}'s of [1], and RA represents the $k^+ \bar{\rho}^{\pm}$'s of [1]. In (27), C1 is either ε_n or $2j$. In (28), C2 is either $4j$ or $2\varepsilon_n$. The sum of the two RA's in the denominator of (28) is due to [1, Eq. (12)]. The complex numbers (27) are t directed currents, either electric or magnetic. The complex numbers (28) are ϕ directed currents. The set of complex numbers (28) is linked to the set

(27) by the fact that the XX's used in (28) occur immediately after those used in (27).

First, the subroutine PRNT (J1, J2, J3, J4, RA) prints out the real part, the imaginary part, and the magnitude of each of the complex numbers (27) under the heading which appears in statement 10. Then, PRNT prints out the real part, the imaginary part, and the magnitude of each of the complex numbers (28) under the heading which appears in statement 13. The variables in (27) and (28) enter PRNT by means of the arguments of PRNT and the statement

```
COMMON C1, C2, XX, JX
```

in line 6. If J1 = 0, nothing from (27) is printed out. If J2 = 0, nothing from (28) is printed out. All the arguments of PRNT are input arguments. The common variables C1, C2, and XX are input variables. However, the common variable JX functions as both an input variable and an output variable. PRNT adds J1 + J2 to the original value of JX.

Minimum allocations in PRNT are given by

```
COMPLEX XX(J2 + J1 + JX)
DIMENSION RA(Max(J1 + J3, J2 + J4))
```

where Max denotes the larger of the two values in the parentheses. Of course, the space allocated to XX in the calling program must be exactly the same as the space allocated to XX in line 4 of PRNT.

DO loop 11 prints out the complex numbers (27). Line 14 obtains (27). DO loop 14 prints out the complex numbers (28). Line 27 obtains (28).

```
C     LISTING OF THE SUBROUTINE PRNT
      SUBROUTINE PRNT(J1.J2.J3.J4.RA)
      COMPLEX C1.C2.U
      COMPLEX XX(79)
      DIMENSION RA(43)
      COMMON C1.C2.XX.JX
      IF(J1.EQ.0) GO TO 15
      WRITE(3.10)
   10 FORMAT('0  REAL JT     IMAG JT      MAG JT')
      K3=J3
      DO 11 J=1.J1
      K3=K3+1
      JX=JX+1
      U=C1/RA(K3)*XX(JX)
      W=CABS(U)
      WRITE(3.12) U.W
   12 FORMAT(1X.3E11.4)
   11 CONTINUE
   15 IF(J2.EQ.0) RETURN
      WRITE(3.13)
   13 FORMAT('0  REAL JP     IMAG JP      MAG JP')
      K4=J4
      DO 14 J=1.J2
      JX=JX+1
      K3=K4
      K4=K4+1
      U=C2/(RA(K3)+RA(K4))*XX(JX)
      W=CABS(U)
      WRITE(3.12) U.W
   14 CONTINUE
      RETURN
      END
```

V. THE MAIN PROGRAM

The main program accepts input data and calls the subroutines YZA, PLANE, DECOMP, SOLVE, and PRNT in order to calculate and print out [1, Eqs. (87)-(89)]. The input data are read from punched cards according to

```
         READ(1,15) NT, NPHI
   15    FORMAT(2I3)
         READ(1,10)(XT(K), K=1, NT)
         READ(1,10)(AT(K), K=1, NT)
   10    FORMAT(5E14.7)
         READ(1,10)(X(K), K=1, NPHI)
         READ(1,10)(A(K), K=1, NPHI)
         READ(1,16) NA, NB, MA, MB, MC, LA, LB, LC,
              M1, M2, BK, UR, ER, THR(1)
   16    FORMAT(10I3/4E14.7)
         READ(1,18)(RA(I), I=1, NA)
         READ(1,18)(ZA(I), I=1, NA)
   18    FORMAT(10F8.4)
         READ(1,18)(RB(I), I=1, NB)
         READ(1,18)(ZB(I), I=1, NB)
```

Most of the input variables in the main program represent variables in [1] and [2]. Table 3 relates input variables in the main program to variables in [1] and [2]. The input variable NT in Table 3 represents both n_t and n_T. It is assumed that $n_t = n_T$ in the main program. The Gaussian quadrature data $x_{\ell}^{(n_t)}$, $A_{\ell}^{(n_t)}$, $x_{\ell}^{(n_\phi)}$, and $A_{\ell}^{(n_\phi)}$ are given in [4, Appendix A]. It is

Table 3. Input data for the main program.

Variable in main program	Variable in [1] or [2]	Equation or figure number in [1] or [2]
NT	$n_t = n_T$	[2, Eqs. (35) and (79)]
NPHI	n_ϕ	[2, Eq. (36)]
XT	$x_{\ell'}^{(n_t)}$, $\ell' = 1,2,\ldots n_t$	[2, Eq. (35)]
AT	$A_{\ell'}^{(n_t)}$, $\ell' = 1,2,\ldots n_t$	[2, Eq. (35)]
X	$x_{\ell}^{(n_\phi)}$, $\ell = 1,2,\ldots n_\phi$	[2, Eq. (37)]
A	$A_{\ell}^{(n_\phi)}$, $\ell = 1,2,\ldots n_\phi$	[2, Eq. (36)]
NA	N^+	[1, Fig. 4]
NB	N^-	[1, Fig. 5]
MA	M^+	[1, Fig. 4]
MB	M^-	[1, Fig. 5]
MC	M	[1, Figs. 4 and 5]
BK	k^+	[1, Eqs. (73)-(84)]
UR	μ^-/μ^+	[1, Fig. 1]
ER	$\varepsilon^-/\varepsilon^+$	[1, Fig. 1]
THR(1)	θ_t	[1, Eq. (92)]
RA	$\bar{\rho}_j^+$, $j = 1,2,\ldots N^+$	[1, Eq. (11)]
ZA	$\bar{z}_j^+$, $j = 1,2,\ldots N^+$	[1, Eq. (11)]
RB	$\bar{\rho}_j^-$, $j = 1,2,\ldots N^-$	[1, Eq. (11)]
ZB	$\bar{z}_j^-$, $j = 1,2,\ldots N^-$	[1, Eq. (11)]

assumed that $N^+ \geq 3$, $N^- \geq 3$, and $M \geq 1$. It is also assumed that either $\begin{Bmatrix} M^+ = 0 \\ M^- = 0 \end{Bmatrix}$ or $\begin{Bmatrix} M^+ > 0 \\ M^- > 0 \end{Bmatrix}$ and that either $\begin{Bmatrix} N^+ - M^+ - M = 0 \\ N^- - M^- - M = 0 \end{Bmatrix}$ or $\begin{Bmatrix} N^+ - M^+ - M > 0 \\ N^- - M^- - M > 0 \end{Bmatrix}$. If $\begin{Bmatrix} M^+ = 0 \\ M^- = 0 \end{Bmatrix}$, then there is no conductor below the aperture as in [1, Fig. 18]. If $\begin{Bmatrix} N^+ - M^+ - M = 0 \\ N^- - M^- - M = 0 \end{Bmatrix}$, then there is no conductor above the aperture as in [1, Fig. 10]. In Table 3, THR(1) is in radians.

The input variables LA, LB, LC, M1, and M2 are not listed in Table 3. The input variables LA, LB, and LC allow for generating curves different from those in [1, Figs. 4 and 5]. LA, LB, and LC are either 0 or 1.

LA = 0 if the generating curve of the surface $(S^+ + A)$ does not close upon itself.

LA = 1 if the generating curve of the surface $(S^+ + A)$ closes upon itself.

LB = 0 if the generating curve of the surface $(S^- + A)$ does not close upon itself.

LB = 1 if the generating curve of the surface $(S^- + A)$ closes upon itself.

LC = 0 if the generating curve of the aperture does not close upon itself.

LC = 1 if the generating curve of the aperture closes upon itself.

For example, LA = 0, LB = 1, and LC = 0 for the generating curves in [1, Figs. 4 and 5]. If LA = 1, it is assumed that $M^+ \geq 1$ and $N^+ - M^+ - M \geq 2$.

If LB = 1, it is assumed that $M^- \geq 1$ and that $N^- - M^- - M \geq 2$. LC = 1 implies that the object under consideration reduces to a homogeneous material obstacle with parameters (μ^-, ϵ^-) in contrast with the parameters of the external environment. As for M1 and M2, [1, Eqs. (87)-(89)] are printed out for n = M1, M1 + 1,...M2.

Minimum allocations in the main program are given by

```
COMPLEX XX(N), YP(M*JA*JA), ZP(M*JA*JA),
                YM(M*JB*JB), ZM(M*JB*JB), R(2*M*JA),
                T(N*N), Y(N)
DIMENSION  XT(NT), AT(NT), X(NPHI), A(NPHI), RA(NA),
             ZA(NA), RB(NB), ZB(NB), IPS(N)
```

where

$$N = \sum_{I=1}^{4} L1A(I) + \sum_{I=1}^{4} L1B(I) + L2(1) + L2(2) + L3(1) + L3(2) \tag{29}$$

$$M = M2 - M1 + 1 \tag{30}$$

$$JA = 2 * NA - 3 \tag{31}$$

$$JB = 2 * NB - 3 \tag{32}$$

The L's on the right-hand side of (29) are calculated in the main program. At any rate, N is the order of the moment matrix T_n given by [1, Eq. (33)].

At the end of this section, the main program is listed along with sample input and output data. The sample data are for the object of [1, Fig. 13] with $k^+a = 2.5$ and $\epsilon_r = 2$ as in [1, Fig. 17]. The output data printed under the heading "Electric current on first part of outside conductor" are, with reference to [1, Eq. (87)],

$$J_n^{t+\theta}(\bar{t}_j^+), \quad j = 2,3,\ldots M^+ \tag{33}$$

and

$$J_n^{\phi+\theta}(t_j^+), \quad j = 1,2,3,\ldots M^+ \tag{34}$$

The real and imaginary parts and magnitudes of (33) are printed under the heading "Real JT Imag JT Mag JT." The real and imaginary parts and magnitudes of (34) are printed under the heading "Real JP Imag JP Mag JP." The output data printed under the heading "Electric current on second part of outside conductor" are

$$J_n^{t+\theta}(\bar{t}_j^+), \quad j = M^+ + M + 1, \ M^+ + M + 2, \ldots N^+ - 1 \tag{35}$$

and

$$J_n^{\phi+\theta}(t_j^+), \quad j = M^+ + M, \ M^+ + M + 1, \ldots N^+ - 1 - LA \tag{36}$$

The output data printed under the heading "Electric current on first part of inside conductor"are, with reference to [1, Eq. (88)],

$$J_n^{t-\theta}(\bar{t}_j^-), \quad j = 2,3,\ldots M^- \tag{37}$$

and

$$J_n^{\phi-\theta}(t_j^-), \quad j = 1,2,3,\ldots M^- \tag{38}$$

The output data printed under the heading "Electric current on second part of inside conductor" are

$$J_n^{t-\theta}(\bar{t}_j^-), \quad j = M^- + M + 1, \ M^- + M + 2, \ldots N^- - 1 \tag{39}$$

and

$$J_n^{\phi-\theta}(t_j^-), \quad j = M^- + M, \ M^- + M + 1, \ldots N^- - 1 - LB \tag{40}$$

The output data printed under the heading "Electric current in aperture" are, with reference to [1, Eq. (87)],

$$J_n^{t+\theta}(\bar{t}_j^+), \quad j = M^+ + 1, \ M^+ + 2, \ldots M^+ + M \tag{41}$$

and

$$J_n^{\phi+\theta}(t_j^+), \quad j = M^+ + 1, \ M^+ + 2, \ldots \ M^+ + M - 1 - LC \tag{42}$$

If, as in [1, Fig. 18], there is no conducting surface below the aperture, then $j = M^{+} + 1$ is deleted in (41). If, as in [1, Fig. 10], there is no conducting surface above the aperture, then $j = M^{+} + M$ is deleted in (41). The output data printed under the heading "Magnetic current in aperture" are, with reference to [1, Eq. (89)],

$$M_n^{t\theta}(\overline{t}_j^{+}), \quad j = M^{+} + 2, \ M^{+} + 3, \ldots M^{+} + M - 1 \tag{43}$$

and

$$M_n^{\phi\theta}(t_j^{+}), \quad j = M^{+} + 1, \ M^{+} + 2, \ldots M^{+} + M - 1 - LC \tag{44}$$

If, in any of (33)-(44), the upper limit on j is less than the starting value of j, then no values of j are to be taken. For example, if $M^{+} = 1$ in (33), then no values of j are to be taken in (33).

Since [1, Eq. (12)] is used for $\rho_j^{\pm}$, the factors 2, 4j, 2j, and 4 are needed in [1, Eqs. (87)-(89)] when $n \geq 1$. In the main program, lines 48-51 store these factors in C3, C4, C5, and C6. Line 53 stores in BB the propagation constant of the medium characterized by (μ^{-}, ϵ^{-}) in [1, Fig. 1]. This propagation constant is called k^{-}. With regard to [1, Eq. (33)], line 54 stores η_r in ET and line 55 stores $1/\eta_r$ in ET1. DO loop 23 puts $k^{+}\overline{\rho}_I^{+}$ in RA(I) and $k^{+}\overline{z}_I^{+}$ in ZA(I). DO loop 24 puts $k^{-}\overline{\rho}_I^{-}$ in RB(I) and $k^{-}\overline{z}_I^{-}$ in ZB(I).

The elements $Y_{nij}^{pq\pm}$ and $Z_{nij}^{pq\pm}$ of the submatrices on the right-hand side of [1, Eq. (33)] are given by [1, Eqs. (36) and (35)]. The testing functions $\underline{W}_{ni}^{p\pm}$ and the expansion functions $\underline{J}_{nj}^{q\pm}$ appear in [1, Eqs. (36) and (35)]. However, the submatrices supplied by the subroutine YZA are $Y_n'^{rs\pm}$ and $Z_n^{rs\pm}$ whose elements are defined by

$$\left.\begin{aligned} Y'^{rs\pm}_{ni'j'} &= -\langle \underline{W}^{r\pm}_{ni'},\ \underline{H}^{\pm}(\underline{J}^{s\pm}_{nj'}, 0)\rangle \qquad (45) \\ Z^{rs\pm}_{ni'j'} &= -\langle \underline{W}^{r\pm}_{ni'},\ \frac{1}{\eta^{\pm}} \underline{E}^{\pm}(\underline{J}^{s\pm}_{nj'}, 0)\rangle \qquad (46) \end{aligned}\right\} \begin{aligned} r &= t,\phi \\ s &= t,\phi \end{aligned}$$

Equations (45) and (46) are (3) and (4) with the superscript $\pm$ appended, with i replaced by i', and with j replaced by j'. In (45) and (46), the testing functions are $\underline{W}^{r\pm}_{ni'}$ and the expansion functions are $\underline{J}^{s\pm}_{nj'}$. To obtain $Y^{pq\pm}_{nij}$ from $Y'^{rs\pm}_{ni'j'}$ and $Z^{pq\pm}_{nij}$ from $Z^{rs\pm}_{ni'j'}$, we must express $\underline{W}^{p\pm}_{ni}$ in terms of $\underline{W}^{r\pm}_{ni'}$ and $\underline{J}^{q\pm}_{nj}$ in terms of $\underline{J}^{s\pm}_{nj'}$. From [1, Eqs. (23), (24), (27), (28), (30), and (37)], we obtain

$$\underline{W}^{p\pm}_{ni} = (\underline{J}^{p\pm}_{ni})^{*} \qquad (47)$$

where * denotes complex conjugate. [1, Eq. (48)] and (47) state that the testing functions are the complex conjugates of the expansion functions. Hence, the expressions for $\underline{W}^{p\pm}_{ni}$ in terms of $\underline{W}^{r\pm}_{ni'}$ will be similar to the expressions for $\underline{J}^{q\pm}_{nj}$ in terms of $\underline{J}^{s\pm}_{nj'}$. Consequently, it suffices to express $\underline{J}^{q\pm}_{nj}$ in terms of $\underline{J}^{s\pm}_{nj'}$.

[1, Eq. (16)] with the choice of superscript + is rewritten as

$$\underline{J}^{1+}_{nj} = \begin{cases} \underline{J}^{+}_{nj'}, \ j' = 1+M1A(1), 2+M1A(1), \ldots L1A(1) + M1A(1) & (48a) \\ \underline{J}^{+}_{nj'}, \ j' = 1+M1A(2), 2+M1A(2), \ldots L1A(2) + M1A(2) & (48b) \\ \underline{J}^{+}_{nj'}, \ j' = 1+M1A(3), 2+M1A(3), \ldots L1A(3) + M1A(3) & (48c) \\ \underline{J}^{+}_{nj'}, \ j' = 1+M1A(4), 2+M1A(4), \ldots L1A(4) + M1A(4) & (48d) \end{cases}$$

where

$$\underline{J}^{+}_{nj'} = \begin{cases} \underline{J}^{t+}_{nj'} , & j' = 1, 2, \ldots N^{+} - 2 \quad (49a) \\ \underline{J}^{\phi+}_{n,j'-N^{+}+2} , & j' = N^{+} - 1, N^{+}, \ldots 2N^{+} - 3 \quad (49b) \end{cases}$$

The vector functions (49a) are the t directed expansion functions on the surface (S^{+} + A) in [1, Fig. 4]. The vector functions (49b) are the ϕ directed expansion functions on the surface (S^{+} + A) in [1, Fig. 4]. Instead of expressing each j' in (48) in terms of j, we state that $\underline{J}^{1+}_{nj}$ is the jth vector function listed on the right-hand side of (48). For example, if j = L1A(1)+3 and if L1A(2) $\geq$ 3, then $\underline{J}^{1+}_{nj} = \underline{J}^{+}_{n,3+M1A(2)}$. In (48),

$$L1A(1) = Max(0, M^{+} - 1) \quad (50a)$$

$$L1A(2) = M^{+} \quad (50b)$$

$$L1A(3) = Max(0, N^{+} - M^{+} - M - 1) \quad (50c)$$

$$L1A(4) = N^{+} - M^{+} - M - LA \quad (50d)$$

$$M1A(1) = 0 \quad (51a)$$

$$M1A(2) = N^{+}-2 \quad (51b)$$

$$M1A(3) = M^{+} + M - 1 \quad (51c)$$

$$M1A(4) = N^{+} + M^{+} + M - 3 \quad (51d)$$

where Max denotes maximum value. The parameters (L1A(I), I=1,2,3,4) and (M1A(I), I=1,2,3,4) are calculated by lines 64-71 of the main program.

The vector functions (48a) are the t directed expansion functions on the first part of the outside conductor. The first part of the outside conductor is the part of S^+ below the aperture in [1, Fig. 4]. The vector functions (48b) are the ϕ directed expansion functions on the first part of the outside conductor. The vector functions (48c) are the t directed expansion functions on the second part of the outside conductor. The second part of the outside conductor is the part of S^+ above the aperture in [1, Fig. 4]. The vector functions (48d) are the ϕ directed expansion functions on the second part of the outside conductor.

$$\underline{J}^{1-}_{nj} = \begin{cases} \underline{J}^{-}_{nj'}, \; j' = 1+M1B(1),\ 2+M1B(1),\ \ldots\ L1B(1) + M1B(1) & \text{(52a)} \\ \underline{J}^{-}_{nj'}, \; j' = 1+M1B(2),\ 2+M1B(2),\ \ldots\ L1B(2) + M1B(2) & \text{(52b)} \\ \underline{J}^{-}_{nj'}, \; j' = 1+M1B(3),\ 2+M1B(3),\ \ldots\ L1B(3) + M1B(3) & \text{(52c)} \\ \underline{J}^{-}_{nj'}, \; j' = 1+M1B(4),\ 2+M1B(4),\ \ldots\ L1B(4) + M1B(4) & \text{(52d)} \end{cases}$$

where

$$\underline{J}^{-}_{nj'} = \begin{cases} \underline{J}^{t-}_{nj'}, & j' = 1,2,\ldots\ N^{-}-2 \quad \text{(53a)} \\ \underline{J}^{\phi-}_{n,j'-N^{-}+2}, & j' = N^{-}-1,\ N^{-},\ldots\ 2N^{-}-3 \quad \text{(53b)} \end{cases}$$

The vector functions (53a) are the t directed expansion functions on the surface (S^- + A) in [1, Fig. 5]. The vector functions (53b) are the ϕ directed expansion functions on the surface (S^- + A) in [1, Fig. 5]. Equation (52) means that $\underline{J}^{1-}_{nj}$ is the jth vector function listed on the right-hand side of (52). In (52),

$$L1B(1) = \mathrm{Max}(0,\ M^{-} - 1) \tag{54a}$$

$$L1B(2) = M^{-} \tag{54b}$$

$$L1B(3) = \mathrm{Max}(0,\ N^{-} - M^{-} - M - 1) \tag{54c}$$

$$L1B(4) = N^{-} - M^{-} - M - LB \tag{54d}$$

$$M1B(1) = 0 \tag{55a}$$

$$M1B(2) = N^{-} - 2 \tag{55b}$$

$$M1B(3) = M^{-} + M - 1 \tag{55c}$$

$$M1B(4) = N^{-} + M^{-} + M - 3 \tag{55d}$$

The parameters (L1B(I), I = 1,2,3,4) and (M1B(I), I = 1,2,3,4) are calculated by lines 72-79 of the main program.

The vector functions (52a) are the t directed expansion functions on the first part of the inside conductor and (52b) are the ϕ directed expansion functions there. The first part of the inside conductor is the part of (S^{-} + A) below the aperture in [1, Fig. 5]. The vector functions (52c) are the t directed expansion functions on the second part of the inside conductor and (52d) are the ϕ directed expansion functions there. The second part of the inside conductor is the part of (S^{-} + A) above the aperture in [1, Fig. 5].

[1, Eq. (17)] with the choice of superscript + is rewritten as

$$\underline{J}_{nj}^{2+} = \begin{cases} \underline{J}_{nj'}^{+}, \quad j' = 1+M2A(1),\ 2+M2A(1),\ \ldots\ L2(1) + M2A(1) & (56a) \\ \\ \underline{J}_{nj'}^{+}, \quad j' = 1+M2A(2),\ 2+M2A(2),\ \ldots\ L2(2) + M2A(2) & (56b) \end{cases}$$

where $\underline{J}_{nj'}^{+}$ is given by (49). Equation (56) means that $\underline{J}_{nj}^{2+}$ is the jth vector function listed on the right-hand side of (56). In (56),

$$L2(1) = \begin{cases} M & M^{+} \neq 0,\ L1A(4) \neq 0 \\ M-1 & M^{+} = 0,\ L1A(4) \neq 0 \\ M-1 & M^{+} \neq 0,\ L1A(4) = 0 \\ M-2 & M^{+} = 0,\ L1A(4) = 0 \end{cases} \tag{57a}$$

$$L2(2) = M - 1 - LC \tag{57b}$$

$$M2A(1) = \mathrm{Max}(0,\ M^{+} - 1) \tag{58a}$$

$$M2A(2) = N^{+} - 2 + M^{+} \tag{58b}$$

where L1A(4) is given by (50d). The parameters L2(1), L2(2), M2A(1), and M2A(2) are calculated by lines 80-85 of the main program.

The vector functions (56a) consist of the t directed expansion function which straddles the first part of the outside conductor and the aperture, the t directed expansion functions in the aperture, and the t directed expansion function which straddles the aperture and the second part of the outside conductor. The vector functions (56b) are the ϕ directed expansion functions in the aperture. Four different cases are necessary in (57a) because there is no t directed expansion function which straddles the first part of the outside conductor and the aperture if the first part of the outside conductor is absent and there is no t directed expansion function which straddles the aperture and the second part of the outside conductor if the second part of the outside conductor is absent.

[1, Eq. (17)] with the choice of superscript - is rewritten as

$$\underline{J}_{nj}^{2-} = \begin{cases} \underline{J}_{nj'}^{-}\ , \quad j' = 1+M2B(1),\ 2+M2B(1),\ \ldots\ L2(1) + M2B(1) & (59a) \\ \\ \underline{J}_{nj'}^{-}\ , \quad j' = 1+M2B(2),\ 2+M2B(2),\ \ldots\ L2(2) + M2B(2) & (59b) \end{cases}$$

where $\underline{J}_{nj'}^{-}$ is given by (53). Equation (59) means that $\underline{J}_{nj}^{2-}$ is the jth vector function listed on the right-hand side of (59). In (59), L2(1) and L2(2) are given by (57). M2B(1) and M2B(2) are given by

$$M2B(1) = Max(0,\ M^{-} - 1) \tag{60a}$$

$$M2B(2) = N^{-} - 2 + M^{-} \tag{60b}$$

M2B(1) and M2B(2) are calculated by lines 86-87 of the main program. The vector functions (59a) consist of the t directed expansion function which straddles the first part of the inside conductor and the aperture, the t directed expansion functions in the aperture, and the t directed expansion function which straddles the aperture and the second part of the inside conductor. The vector functions (59b) are the ϕ directed expansion functions in the aperture. Because of [1, Eqs. (15b) and (15d)], the vector functions in the aperture in (59) are the same as the vector functions in the aperture in (56).

[1, Eq. (22)] is rewritten as

$$\underline{J}^{3+}_{nj} = \begin{cases} \underline{J}^{+}_{nj'}\ ,\ j' = 1{+}M3A(1),\ 2{+}M3A(1),\ \ldots\ L3(1) + M3A(1) & \text{(61a)} \\[2em] \underline{J}^{+}_{nj'}\ ,\ j' = 1{+}M3A(2),\ 2{+}M3A(2),\ \ldots\ L3(2) + M3A(2) & \text{(61b)} \end{cases}$$

where $\underline{J}^{+}_{nj'}$ is given by (49). Equation (61) means that $\underline{J}^{3+}_{nj}$ is the jth vector function listed on the right-hand side of (61). In (61),

$$L3(1) = Max(0,\ M - 2) \tag{62a}$$

$$L3(2) = L2(2) \tag{62b}$$

$$M3A(1) = M^{+} \tag{63a}$$

$$M3A(2) = N^{+} - 2 + M^{+} \tag{63b}$$

where L2(2) is given by (57b). L3(1), L3(2), M3A(1), and M3A(2) are calculated by lines 88-91 of the main program. The vector functions (61a) are the t directed expansion functions in the aperture. The vector functions (61b) are the ϕ directed expansion functions in the aperture.

Replacement of the superscript + in [1, Eq. (22)] by the superscript - gives

$$\underline{J}_{nj}^{3-} = \begin{cases} \underline{J}_{n,j+M^-}^{t-} & , \ j = 1,2, \ldots M-2 \\ \underline{J}_{n,j+M^- - M+2}^{\phi-} & , \ j = M-1, M, \ldots 2M-3 \end{cases} \tag{64}$$

Equation (64) is consistent with [1, Eqs. (22), (37), (15b), and (15d)]. Equation (64) can be rewritten as

$$\underline{J}_{nj}^{3-} = \begin{cases} \underline{J}_{nj'}^{-} \ , \ j' = 1+M3B(1), 2+M3B(1), \ldots L3(1) + M3B(1) & (65a) \\ \underline{J}_{nj'}^{-} \ , \ j' = 1+M3B(2), 2+M3B(2), \ldots L3(2) + M3B(2) & (65b) \end{cases}$$

where $\underline{J}_{nj'}^{-}$ is given by (53). Equation (65) means that $\underline{J}_{nj}^{3-}$ is the jth vector function listed on the right-hand side of (65). In (65), L3(1) and L3(2) are given by (62a) and (62b). M3B(1) and M3B(2) are given by

$$M3B(1) = M^- \tag{66a}$$

$$M3B(2) = N^- - 2 + M^- \tag{66b}$$

M3B(1) and M3B(2) are calculated by lines 92-93 of the main program. The vector functions on the right-hand side of (65) are the same as those on the right-hand side of (61).

To accompany the expansion functions $\underline{J}_{nj'}^{\pm}$ of (49) and (53), testing functions $\underline{W}_{ni'}^{\pm}$ are defined by

$$\underline{W}^{\pm}_{ni'} = (\underline{J}^{\pm}_{ni'})^* \tag{67}$$

Equations (49), (53), and (67) allow (45) and (46) to be recast as

$$Y'^{\pm}_{ni'j'} = - \langle \underline{W}^{\pm}_{ni'} , H^{\pm}(\underline{J}^{\pm}_{nj'} , 0)\rangle \tag{68}$$

$$Z^{\pm}_{ni'j'} = - \langle \underline{W}^{\pm}_{ni'} , \frac{1}{\eta^{\pm}} E^{\pm}(\underline{J}^{\pm}_{nj'} , 0)\rangle \tag{69}$$

Expressions (68) and (69) are the matrix elements supplied by the subroutine YZA.

Line 94 stores in JA the order of the matrices Y'^{+}_{n} and Z^{+}_{n} whose i'j'th elements are given by (68) and (69). Line 95 stores in JB the order of the matrices Y'^{-}_{n} and Z^{-}_{n} whose i'j'th elements are given by (68) and (69). Line 96 stores in IA the total number of vector functions listed on the right-hand side of (48). IA is the order of the submatrix Z^{11+}_{n} in [1, Eq. (33)]. Line 97 stores in IB the total number of vector functions listed on the right-hand side of (52). IB is the order of the submatrix Z^{11-}_{n} in [1, Eq. (33)]. Line 98 stores in N the order of the matrix T_n of [1, Eq. (33)].

Line 99 puts Y'^{+}_{n} of (68) in YP ((n-M1)*JA*JA+1) to YP((n-M1+1)*JA*JA) and Z^{+}_{n} of (69) in the corresponding region of ZP for n = M1, M1+1, ... M2. Storage of Y'^{+}_{n} and Z^{+}_{n} in YP and ZP is by columns. Line 100 puts Y'^{-}_{n} of (68) in YM((n-M1)*JB*JB+1) to YM((n-M1+1)*JB*JB) and Z^{-}_{n} of (69) in the corresponding region of ZM for n = M1, M1+1, ... M2. Storage of Y'^{-}_{n} and Z^{-}_{n} in YM and ZM is by columns. With reference to [1, Eq. (103)], line 101 puts

$$\left.\begin{array}{llll} V^{t\theta}_{ni'} & \text{in} & R(i' + J1) & , \quad i' = 1,2,\ldots NA-2 \\ -V^{\phi\theta}_{ni'} & \text{in} & R(i' + J1 + NA - 2) & , \quad i' = 1,2,\ldots NA-1 \\ -V^{t\phi}_{ni'} & \text{in} & R(i' + J1 + JA) & , \quad i' = 1,2,\ldots NA-2 \\ V^{\phi\phi}_{ni'} & \text{in} & R(i' + J1 + JA + NA-2) & , \quad i' = 1,2,\ldots NA-1 \end{array}\right\} \quad (70)$$

where

$$J1 = (n - M1) * 2 * JA \quad (71)$$

and

$$n = M1,\ M1 + 1,\ \ldots\ M2 \quad (72)$$

DO loop 82 puts $k^{+}\bar{\rho}^{-}_{J}$ in RB(J). $k^{+}\bar{\rho}^{-}_{J}$ is needed in order to calculate [1, Eq. (88a)].

DO loop 86 obtains n according to n = M-1. If n ≠ M1, DO loop 89 moves Y'^{+}_{n} down into YP(1) to YP(JA*JA) and Z^{+}_{n} down into ZP(1) to ZP(JA*JA), DO loop 90 moves Y'^{-}_{n} down into YM(1) to YM(JB*JB) and Z^{-}_{n} down into ZM(1) to ZM(JB*JB), and DO loop 91 moves the V's of (70) down into R(1) to R(2*JA).

The only difference between $Y^{pq\pm}_{nij}$ of [1, Eq. (36)] and $Y'^{\pm}_{ni'j'}$ of (68) lies in the testing functions and expansion functions. Likewise, the only difference between $Z^{pq\pm}_{nij}$ of [1, Eq. (35)] and $Z^{\pm}_{ni'j'}$ of (69) lies in the testing functions and expansion functions. The expansion functions in [1, Eqs. (36) and (35)] are related to the expansion functions in (68) and (69) by (48), (52), (56), (59), (61), and (65). According to (67) and (47), the testing functions are the complex conjugates of the expansion functions. Hence, the complex conjugates of (48), (52), (56), (59), (61), and (65) relate the testing functions in [1, Eqs. (36) and (35)] to the testing functions in (68) and (69). As a result,

$$Y_{nij}^{pq\pm} = Y_{ni'j'}^{'\pm} \tag{73}$$

$$Z_{nij}^{pq\pm} = Z_{ni'j'}^{\pm} \tag{74}$$

where j' is the subscript of the jth vector function listed on the right-hand side of either (48), (52), (56), (59), (61), or (65), whichever is appropriate. Similarly, i' is the subscript of the ith vector function listed on the right-hand side of either (48), (52), (56), (59), (61), or (65), whichever is appropriate. For example, if $p = 2$ and $q = 1$ and if the superscript + is chosen in (73), then (73) becomes

$$Y_{nij}^{21+} = Y_{ni'j'}^{'+} \tag{75}$$

where j' is the subscript of the jth vector function listed on the right-hand side of (48) and i' is the subscript of the ith vector function listed on the right-hand side of (56).

With regard to [1, Eq. (33)], DO loop 25 uses (73) and (74) to store

$$\begin{bmatrix} Z_n^{11+} \\ 0 \\ Z_n^{21+} \\ Y_n^{31+} \end{bmatrix} \tag{76}$$

by columns in T. The elements of the submatrices in (76) depend on the expansion functions (48). Hence, j' in (73) and (74) is the subscript of the jth vector function listed on the right-hand side of (48). The index of DO loop 25 is JJ. If JJ = 1, inner DO loop 26 obtains the values of j for which j' is given by (48a). If JJ = 2, DO loop 26 obtains the values of j for which j' is given by (48b). Similarly, if JJ = 3, DO loop 26 obtains the values of j covered by (48c). Finally, if JJ = 4, DO loop 26 obtains the values of j covered by (48d).

The elements of Z_n^{11+} in (76) depend on the testing functions $\underline{W}_{ni}^{1+}$. These testing functions are related to $\underline{W}_{ni'}^{+}$ of (67) by the testing function version of (48). The testing function version of (48) is (48) with $\underline{J}_{nj}^{1+}$ replaced by $\underline{W}_{ni}^{1+}$, $\underline{J}_{nj'}^{+}$ replaced by $\underline{W}_{ni'}^{+}$, and j' replaced by i'. It is now apparent that i' in (74) is the subscript of the ith vector function listed on the right-hand side of the testing function version of (48). Inside nested DO loops 27 and 28, line 141 stores the appropriate element of Z_n^{11+} of (76) in T. The index of DO loop 27 is II. If II = 1, DO loop 28 obtains the values of i for which i' is given by the testing function version of (48a). If II = 2, DO loop 28 obtains the values of i for which i' is given by the testing function version of (48b). Similarly, if II = 3, DO loop 28 obtains the values of i covered by the testing function version of (48c). Finally, if II = 4, DO loop 28 obtains the values of i covered by the testing function version of (48d).

DO loop 29 takes care of the 0 in (76).

The elements of Z_n^{21+} in (76) depend on the testing functions $\underline{W}_{ni}^{2+}$. These testing functions are related to $\underline{W}_{ni'}^{+}$ of (67) by the testing function version of (56). Hence, i' in (74) is the subscript of the ith vector function listed on the right-hand side of the testing function version of (56). Inside nested DO loops 30 and 31, line 157 stores the appropriate element of Z_n^{21+} of (76) in T. The index of DO loop 30 is II. If II = 1, DO loop 31 obtains the values of i for which i' is given by the testing function version of (56a). If II = 2, DO loop 31 obtains the values of i for which i' is given by the testing function version of (56b).

The elements of Y_n^{31+} in (76) depend on the testing functions $\underline{W}_{ni}^{3+}$. These testing functions are related to $\underline{W}_{ni'}^{+}$, of (67) by the testing function version of (61). Hence i' in (73) is the subscript of the ith vector function listed on the right-hand side of the testing function version of (61). Inside nested DO loops 32 and 33, line 168 stores the appropriate element of Y_n^{31+} of (76) in T. The index of DO loop 32 is II. If II = 1, DO loop 33 obtains the values of i for which i' is given by the testing function version of (61a). If II = 2, DO loop 33 obtains the values of i for which i' is given by the testing function version of (61b).

With regard to [1, Eq. (33)], DO loop 34 uses (73) and (74) to store

$$\begin{bmatrix} 0 \\ \eta_r Z_n^{11-} \\ \eta_r Z_n^{21-} \\ Y_n^{31-} \end{bmatrix} . \tag{77}$$

by columns in T. Nested DO loops 34 and 35 obtain the values of j in (73) and (74). In (73) and (74), j' is the subscript of the jth vector function listed on the right-hand side of (52). DO loop 36 obtains the 0 in (77). Nested DO loops 37 and 38 obtain the values of i for Z_{nij}^{11-} of (74) in which i' is the subscript of the ith vector function listed on the right-hand side of the testing function version of (52). Nested DO loops 39 and 40 obtain the values of i for Z_{nij}^{21-} of (74) in which i' is determined by the testing function version of (59). Nested DO loops 41 and 42 obtain the values of i for

Y_{nij}^{31-} of (73) in which i' is determined by the testing function version of (65).

With regard to [1, Eq. (33)], DO loop 43 uses (73) and (74) to store

$$\begin{bmatrix} Z_n^{12+} \\ \eta_r \, Z_n^{12-} \\ Z_n^{22+} + \eta_r \, Z_n^{22-} \\ Y_n^{32+} + \quad Y_n^{32-} \end{bmatrix} \tag{78}$$

by columns in T. Nested DO loops 43 and 44 obtain the values of j in (73) and (74) in which j' is determined by (56) for the submatrices with superscript + in (78) and by (59) for the submatrices with superscript - in (78). Nested DO loops 45 and 46 obtain the values of i for Z_{nij}^{12+} of (74) with i' determined by the testing function version of (48). Nested DO loops 47 and 48 obtain the values of i for Z_{nij}^{12-} of (74) with i' determined by the testing function version of (52). Nested DO loops 49 and 50 obtain the values of i for $Z_{nij}^{22+} + \eta_r \, Z_{nij}^{22-}$ of (78). Z_{nij}^{22+} is given by (74) with i' determined by the testing function version of (56). Z_{nij}^{22-} is given by (74) with i' determined by the testing function version of (59). Nested DO loops 51 and 52 obtain the values of i for $Y_{nij}^{32+} + Y_{nij}^{32-}$ of (78). Y_{nij}^{32+} is given by (73) with i' determined by the testing function version of (61). Y_{nij}^{32-} is given by (73) with i' determined by the testing function version of (65).

With regard to [1, Eq. (33)], DO loop 53 uses (73) and (74) to store

$$\begin{bmatrix} -Y_n^{13+} \\ -Y_n^{13-} \\ -Y_n^{23+} - Y_n^{23-} \\ Z_n^{33+} + (\frac{1}{\eta_r})\, Z_n^{33-} \end{bmatrix} \tag{79}$$

by columns in T. Nested DO loops 53 and 54 obtain the values of j in (73) and (74) with j' determined by (61) for the submatrices with superscript + in (79) and by (65) for the submatrices with superscript - in (79). Nested DO loops 55 and 56 obtain the values of i for Y_{nij}^{13+} of (73) with i' determined by the testing function version of (48). Nested DO loops 57 and 58 obtain the values of i for Y_{nij}^{13-} of (73) with i' determined by the testing function version of (52). Nested DO loops 59 and 60 obtain the values of i for $-Y_{nij}^{23+} - Y_{nij}^{23-}$ of (79). Y_{nij}^{23+} is given by (73) with i' determined by the testing function version of (56). Y_{nij}^{23-} is given by (73) with i' determined by the testing function version of (59). Nested DO loops 61 and 62 obtain the values of i for $Z_{nij}^{33+} + (\frac{1}{\eta_r})\, Z_{nij}^{33-}$ of (79). Z_{nij}^{33+} is given by (74) with i' determined by the testing function version of (61). Z_{nij}^{33-} is given by (74) with i' determined by the testing function version of (65).

Now that the moment matrix T_n has been stored in T, we turn to the excitation vector $\tilde{B}_n^{\theta}$ of [1, Eq. (101)]. The V's on the right-hand side of [1, Eq. (103)] reside in R. Because DO loop 91 has been executed, storage in R is not according to (70) as it stands but according to (70) without the offset J1. The auxiliary equation involving the W's in

[1, Eq. (103a)] is the testing function version of (48). Likewise, the auxiliary equation in [1, Eq. (103b)] is the testing function version of (56), and the auxiliary equation in [1, Eq. (103c)] is the testing function version of (61).

Based on the considerations in the preceding paragraph, lines 336-378 store $\tilde{B}_n^{\theta}$ of [1, Eq. (101)] in Y. Nested DO loops 63 and 64 store $\tilde{V}_n^{i1+\theta}$ of [1, Eq. (101)] in Y. The sign factor S in line 346 compensates for the minus sign in front of $V_{ni'}^{\phi\theta}$ in (70). The number of elements in the row vector 0 in [1, Eq. (101)] is IB because this row vector can be traced back to [1, Eq. (39b)]. DO loop 65 stores this row vector in Y. Nested DO loops 76 and 77 store $\tilde{V}_n^{i\theta}$ of [1, Eq. (101)] in Y. Line 364 compensates for the minus sign in front of $V_{ni'}^{\phi\theta}$ in (70). Nested DO loops 66 and 67 store $\tilde{I}_n^{i\theta}$ of [1, Eq. (101)] in Y. The offset JA in line 370 is mandated by the superscript ϕ in $V_{ni'}^{r\phi}$ in [1, Eq. (103c)]. Line 376 accounts for the net effect of the minus sign in front of $V_{ni'}^{r\phi}$ in [1, Eq. (103c)] and the minus sign in front of $V_{ni'}^{t\phi}$ in (70).

Line 379 decomposes T_n into the product of a lower triangular matrix with an upper triangular matrix. Line 380 stores the solution $\vec{X}_n^{\theta}$ to [1, Eq. (65a)] in XX. The elements [1, Eq. (68a)] of $\tilde{X}_n^{\theta}$ are the I's and V's in [1, Eqs. (87)-(89)]. Lines 384-385 store ε_n in C1. Line 386 stores 4j in C2. If L1A(2) = 0, then it is evident (50a) and (50b) that L1A(1) = 0 so that no values of j are to be taken in (33) and (34). If L1A(2) > 0, line 392 prints (33) and (34). If L1A(4) = 0, then it is evident from (50c) and (50d) that L1A(3) = 0 so that no values of j are to be taken in (35) and (36). If L1A(4) > 0, line 398 prints (35) and (36). Line 403 prints (37) and (38). Line 409 prints (39) and (40).

If L2(1) = L2(2) = 0, then it is evident from (57a), (57b), (62a), and (62b) that L3(1) = L3(2) = 0 so that no values of j are to be taken in (41) - (44). If either L2(1) > 0 or L2(2) > 0, then line 417 prints (41)-(42). If L2(2) = 0, then it is evident from (57b), (62a) and (62b) that L3(1) = L3(2) = 0 so that no values of j are to be taken in (43) and (44). If L2(2) > 0, line 421 puts 2j in C1, lines 422 and 423 put $2\varepsilon_n$ in C2, and line 424 prints (43) and (44).

```
C      LISTING OF THE MAIN PROGRAM
C      THE SUBPROGRAMS YZA.BLOG.PLANE.DECOMP.SOLVE. AND PRNT ARE NEEDED
//PGM JOB (XXXX.XXXX.2.2).'MAUTZ.JOE'.REGION=275K
// EXEC WATFIV
//GO.SYSIN DD *
$JOB              MAUTZ.TIME=5.PAGES=60
      COMPLEX C1.C2.C3.C4.C5.C6.XX(79).YP(2209).ZP(2209).YM(2209)
      COMPLEX ZM(2209).R(240).T(6241).Y(79)
      DIMENSION XT(10).AT(10).X(48).A(48).THR(3).RA(43).ZA(43).RB(43)
      DIMENSION ZB(43).L1A(4).M1A(4).L1B(4).M1B(4).L2(2).M2A(2).M2B(2)
      DIMENSION L3(2).M3A(2).M3B(2).IPS(79)
      COMMON C1.C2.XX.JX
      READ(1.15) NT.NPHI
   15 FORMAT(2I3)
      WRITE(3.9) NT.NPHI
    9 FORMAT('  NT NPHI'/1X.I3.I5)
      READ(1.10)(XT(K).K=1.NT)
      READ(1.10)(AT(K).K=1.NT)
   10 FORMAT(5E14.7)
      WRITE(3.11)(XT(K).K=1.NT)
      WRITE(3.12)(AT(K).K=1.NT)
   11 FORMAT(' XT'/(1X.5E14.7))
   12 FORMAT(' AT'/(1X.5E14.7))
      READ(1.10)(X(K).K=1.NPHI)
      READ(1.10)(A(K).K=1.NPHI)
      WRITE(3.13)(X(K).K=1.NPHI)
      WRITE(3.14)(A(K).K=1.NPHI)
   13 FORMAT(' X'/(1X.5E14.7))
   14 FORMAT(' A'/(1X.5E14.7))
      READ(1.16) NA.NB.MA.MB.MC.LA.LB.LC.M1.M2.BK.UR.ER.THR(1)
   16 FORMAT(10I3/4E14.7)
      WRITE(3.17) NA.NB.MA.MB.MC.LA.LB.LC.M1.M2.BK.UR.ER.THR(1)
   17 FORMAT('  NA NB MA MB MC LA LB LC M1 M2'/1X.10I3/7X.'BK'.12X.'UR'.
     112X.'ER'.10X.'THR(1)'/1X.4E14.7)
      READ(1.18)(RA(I).I=1.NA)
      READ(1.18)(ZA(I).I=1.NA)
   18 FORMAT(10F8.4)
      WRITE(3.19)(RA(I).I=1.NA)
      WRITE(3.20)(ZA(I).I=1.NA)
   19 FORMAT(' RA'/(1X.10F8.4))
   20 FORMAT(' ZA'/(1X.10F8.4))
      READ(1.18)(RB(I).I=1.NB)
      READ(1.18)(ZB(I).I=1.NB)
      WRITE(3.21)(RB(I).I=1.NB)
   21 FORMAT(' RB'/(1X.10F8.4))
      WRITE(3.22)(ZB(I).I=1.NB)
   22 FORMAT(' ZB'/(1X.10F8.4))
      C3=2.
      C4=4.*(0..1.)
      C5=2.*(0..1.)
      C6=4.
      UE=SQRT(UR*ER)
      BB=BK*UE
      ET=SQRT(UR/ER)
      ET1=1./ET
      DO 23 I=1.NA
      RA(I)=BK*RA(I)
      ZA(I)=BK*ZA(I)
   23 CONTINUE
      DO 24 I=1.NB
```

```
      RB(I)=BB*RB(I)
      ZB(I)=BB*ZB(I)
   24 CONTINUE
      L1A(1)=MAX0(0,MA-1)
      L1A(2)=MA
      L1A(3)=MAX0(0,NA-MA-MC-1)
      L1A(4)=NA-MA-MC-LA
      M1A(1)=0
      M1A(2)=NA-2
      M1A(3)=MA+MC-1
      M1A(4)=NA+MA+MC-3
      L1B(1)=MAX0(0,MB-1)
      L1B(2)=MB
      L1B(3)=MAX0(0,NB-MB-MC-1)
      L1B(4)=NB-MB-MC-LB
      M1B(1)=0
      M1B(2)=NB-2
      M1B(3)=MB+MC-1
      M1B(4)=NB+MB+MC-3
      L2(1)=MC
      IF(MA.EQ.0) L2(1)=MC-1
      IF(L1A(4).EQ.0) L2(1)=L2(1)-1
      L2(2)=MC-1-LC
      M2A(1)=MAX0(0,MA-1)
      M2A(2)=NA-2+MA
      M2B(1)=MAX0(0,MB-1)
      M2B(2)=NB-2+MB
      L3(1)=MAX0(0,MC-2)
      L3(2)=L2(2)
      M3A(1)=MA
      M3A(2)=NA-2+MA
      M3B(1)=MB
      M3B(2)=NB-2+MB
      JA=2*NA-3
      JB=2*NB-3
      IA=L1A(1)+L1A(2)+L1A(3)+L1A(4)
      IB=L1B(1)+L1B(2)+L1B(3)+L1B(4)
      N=IA+IB+L2(1)+L2(2)+L3(1)+L3(2)
      CALL YZA(M1,M2,NA,NPHI,NT,0,RA,ZA,X,A,XT,AT,YP,ZP)
      CALL YZA(M1,M2,NB,NPHI,NT,0,RB,ZB,X,A,XT,AT,YM,ZM)
      CALL PLANE(M1,M2,1,NA,NT,RA,ZA,XT,AT,THR,R)
      DO 82 J=1,NB
      RB(J)=RB(J)/UE
   82 CONTINUE
      JA2=JA*JA
      JB2=JB*JB
      M3=M1+1
      M4=M2+1
      DO 86 M=M3,M4
      IF(M.EQ.M3) GO TO 88
      J2=M-M3
      J1=J2*JA2
      DO 89 J=1,JA2
      YP(J)=YP(J+J1)
      ZP(J)=ZP(J+J1)
   89 CONTINUE
      J1=J2*JB2
      DO 90 J=1,JB2
      YM(J)=YM(J+J1)
      ZM(J)=ZM(J+J1)
```

```
   90 CONTINUE
      J3=2*JA
      J1=J2*J3
      DO 91 J=1,J3
      R(J)=R(J+J1)
   91 CONTINUE
   88 JT=0
      DO 25 JJ=1,4
      J2=L1A(JJ)
      IF(J2.EQ.0) GO TO 25
      J1=M1A(JJ)*JA
      DO 26 J=1,J2
      DO 27 II=1,4
      I3=L1A(II)
      IF(I3.EQ.0) GO TO 27
      J3=J1+M1A(II)
      I1=J3+1
      I2=J3+I3
      DO 28 I=I1,I2
      JT=JT+1
      T(JT)=ZP(I)
   28 CONTINUE
   27 CONTINUE
      IF(IB.EQ.0) GO TO 79
      DO 29 I=1,IB
      JT=JT+1
      T(JT)=0.
   29 CONTINUE
   79 DO 30 II=1,2
      I3=L2(II)
      IF(I3.EQ.0) GO TO 30
      J3=J1+M2A(II)
      I1=J3+1
      I2=J3+I3
      DO 31 I=I1,I2
      JT=JT+1
      T(JT)=ZP(I)
   31 CONTINUE
   30 CONTINUE
      DO 32 II=1,2
      I3=L3(II)
      IF(I3.EQ.0) GO TO 32
      J3=J1+M3A(II)
      I1=J3+1
      I2=J3+I3
      DO 33 I=I1,I2
      JT=JT+1
      T(JT)=YP(I)
   33 CONTINUE
   32 CONTINUE
      J1=J1+JA
   26 CONTINUE
   25 CONTINUE
      DO 34 JJ=1,4
      J2=L1B(JJ)
      IF(J2.EQ.0) GO TO 34
      J1=M1B(JJ)*JB
      DO 35 J=1,J2
      IF(IA.EQ.0) GO TO 80
      DO 36 I=1,IA
```

```
      JT=JT+1
      T(JT)=0.
   36 CONTINUE
   80 DO 37 II=1,4
      I3=L1B(II)
      IF(I3.EQ.0) GO TO 37
      J3=J1+M1B(II)
      I1=J3+1
      I2=J3+I3
      DO 38 I=I1,I2
      JT=JT+1
      T(JT)=ET*ZN(I)
   38 CONTINUE
   37 CONTINUE
      DO 39 II=1,2
      I3=L2(II)
      IF(I3.EQ.0) GO TO 39
      J3=J1+M2B(II)
      I1=J3+1
      I2=J3+I3
      DO 40 I=I1,I2
      JT=JT+1
      T(JT)=ET*ZN(I)
   40 CONTINUE
   39 CONTINUE
      DO 41 II=1,2
      I3=L3(II)
      IF(I3.EQ.0) GO TO 41
      J3=J1+M3B(II)
      I1=J3+1
      I2=J3+I3
      DO 42 I=I1,I2
      JT=JT+1
      T(JT)=YN(I)
   42 CONTINUE
   41 CONTINUE
      J1=J1+JB
   35 CONTINUE
   34 CONTINUE
      DO 43 JJ=1,2
      J2=L2(JJ)
      IF(J2.EQ.0) GO TO 43
      J1A=M2A(JJ)*JA
      J1B=M2B(JJ)*JB
      DO 44 J=1,J2
      DO 45 II=1,4
      I3=L1A(II)
      IF(I3.EQ.0) GO TO 45
      J3=J1A+M1A(II)
      I1=J3+1
      I2=J3+I3
      DO 46 I=I1,I2
      JT=JT+1
      T(JT)=ZP(I)
   46 CONTINUE
   45 CONTINUE
      DO 47 II=1,4
      I3=L1B(II)
      IF(I3.EQ.0) GO TO 47
      J3=J1B+M1B(II)
```

```
      I1=J3+1
      I2=J3+I3
      DO 48 I=I1,I2
      JT=JT+1
      T(JT)=ET*ZM(I)
   48 CONTINUE
   47 CONTINUE
      DO 49 II=1,2
      I3=L2(II)
      IF(I3.EQ.0) GO TO 49
      J3=J1A+M2A(II)
      I1=J3+1
      I2=J3+I3
      J4=J1B+M2B(II)
      DO 50 I=I1,I2
      J4=J4+1
      JT=JT+1
      T(JT)=ZP(I)+ET*ZM(J4)
   50 CONTINUE
   49 CONTINUE
      DO 51 II=1,2
      I3=L3(II)
      IF(I3.EQ.0) GO TO 51
      J3=J1A+M3A(II)
      I1=J3+1
      I2=J3+I3
      J4=J1B+M3B(II)
      DO 52 I=I1,I2
      J4=J4+1
      JT=JT+1
      T(JT)=YP(I)+YM(J4)
   52 CONTINUE
   51 CONTINUE
      J1A=J1A+JA
      J1B=J1B+JB
   44 CONTINUE
   43 CONTINUE
      DO 53 JJ=1,2
      J2=L3(JJ)
      IF(J2.EQ.0) GO TO 53
      J1A=M3A(JJ)*JA
      J1B=M3B(JJ)*JB
      DO 54 J=1,J2
      DO 55 II=1,4
      I3=L1A(II)
      IF(I3.EQ.0) GO TO 55
      J3=J1A+M1A(II)
      I1=J3+1
      I2=J3+I3
      DO 56 I=I1,I2
      JT=JT+1
      T(JT)=-YP(I)
   56 CONTINUE
   55 CONTINUE
      DO 57 II=1,4
      I3=L1B(II)
      IF(I3.EQ.0) GO TO 57
      J3=J1B+M1B(II)
      I1=J3+1
      I2=J3+I3
```

```
      DO 58 I=I1,I2
      JT=JT+1
      T(JT)=-YM(I)
   58 CONTINUE
   57 CONTINUE
      DO 59 II=1,2
      I3=L2(II)
      IF(I3.EQ.0) GO TO 59
      J3=J1A+M2A(II)
      I1=J3+1
      I2=J3+I3
      J4=J1B+M2B(II)
      DO 60 I=I1,I2
      J4=J4+1
      JT=JT+1
      T(JT)=-YP(I)-YM(J4)
   60 CONTINUE
   59 CONTINUE
      DO 61 II=1,2
      I3=L3(II)
      IF(I3.EQ.0) GO TO 61
      J3=J1A+M3A(II)
      I1=J3+1
      I2=J3+I3
      J4=J1B+M3B(II)
      DO 62 I=I1,I2
      J4=J4+1
      JT=JT+1
      T(JT)=ZP(I)+ET1*ZM(J4)
   62 CONTINUE
   61 CONTINUE
      J1A=J1A+JA
      J1B=J1B+JB
   54 CONTINUE
   53 CONTINUE
      JY=0
      S=1.
      DO 63 II=1,4
      I3=L1A(II)
      IF(I3.EQ.0) GO TO 83
      J3=M1A(II)
      I1=J3+1
      I2=J3+I3
      DO 64 I=I1,I2
      JY=JY+1
      Y(JY)=S*R(I)
   64 CONTINUE
   83 S=-S
   63 CONTINUE
      IF(IB.EQ.0) GO TO 81
      DO 65 I=1,IB
      JY=JY+1
      Y(JY)=0.
   65 CONTINUE
   81 DO 76 II=1,2
      I3=L2(II)
      IF(I3.EQ.0) GO TO 76
      J3=M2A(II)
      I1=J3+1
      I2=J3+I3
```

```
      DO 77 I=I1,I2
      JY=JY+1
      Y(JY)=R(I)
      IF(II.EQ.2) Y(JY)=-Y(JY)
   77 CONTINUE
   76 CONTINUE
      DO 66 II=1,2
      I3=L3(II)
      IF(I3.EQ.0) GO TO 66
      J3=M3A(II)+JA
      I1=J3+1
      I2=J3+I3
      DO 67 I=I1,I2
      JY=JY+1
      Y(JY)=R(I)
      IF(II.EQ.2) Y(JY)=-Y(JY)
   67 CONTINUE
   66 CONTINUE
      CALL DECOMP(N,IPS,T)
      CALL SOLVE(N,IPS,T,Y,XX)
      MS=M-1
      WRITE(3,96) MS
   96 FORMAT('0',I3,'TH MODE ELECTRIC AND MAGNETIC CURRENTS')
      C1=C3
      IF(MS.EQ.0) C1=1.
      C2=C4
      JX=0
      J2=L1A(2)
      IF(J2.EQ.0) GO TO 84
      WRITE(3,68)
   68 FORMAT('0ELECTRIC CURRENT ON FIRST PART OF OUTSIDE CONDUCTOR')
      CALL PRNT(L1A(1),J2,1,1,RA)
   84 J2=L1A(4)
      IF(J2.EQ.0) GO TO 74
      WRITE(3,69)
   69 FORMAT('0ELECTRIC CURRENT ON SECOND PART OF OUTSIDE CONDUCTOR')
      J3=MA+MC
      CALL PRNT(L1A(3),J2,J3,J3,RA)
   74 J2=L1B(2)
      IF(J2.EQ.0) GO TO 85
      WRITE(3,70)
   70 FORMAT('0ELECTRIC CURRENT ON FIRST PART OF INSIDE CONDUCTOR')
      CALL PRNT(L1B(1),J2,1,1,RB)
   85 J2=L1B(4)
      IF(J2.EQ.0) GO TO 75
      WRITE(3,71)
   71 FORMAT('0ELECTRIC CURRENT ON SECOND PART OF INSIDE CONDUCTOR')
      J3=MB+MC
      CALL PRNT(L1B(3),J2,J3,J3,RB)
   75 J1=L2(1)
      J2=L2(2)
      IF((J1+J2).EQ.0) GO TO 86
      WRITE(3,72)
   72 FORMAT('0ELECTRIC CURRENT IN APERTURE')
      J3=M2A(1)+1
      J4=MA+1
      CALL PRNT(J1,J2,J3,J4,RA)
      IF(J2.EQ.0) GO TO 86
      WRITE(3,73)
   73 FORMAT('0MAGNETIC CURRENT IN APERTURE')
```

```
      C1=C5
      C2=C6
      IF(M5.EQ.0) C2=2.
      CALL PRNT(L3(1),J2,J4,J4,RA)
   86 CONTINUE
      STOP
      END
$DATA
  2 20
-0.5773503E+00 0.5773503E+00
 0.1000000E+01 0.1000000E+01
-0.9931286E+00-0.9639719E+00-0.9122344E+00-0.8391170E+00-0.7463319E+00
-0.6360537E+00-0.5108670E+00-0.3737061E+00-0.2277859E+00-0.7652652E-01
 0.7652652E-01 0.2277859E+00 0.3737061E+00 0.5108670E+00 0.6360537E+00
 0.7463319E+00 0.8391170E+00 0.9122344E+00 0.9639719E+00 0.9931286E+00
 0.1761401E-01 0.4060143E-01 0.6267205E-01 0.8327674E-01 0.1019301E+00
 0.1181945E+00 0.1316886E+00 0.1420961E+00 0.1491730E+00 0.1527534E+00
 0.1527534E+00 0.1491730E+00 0.1420961E+00 0.1316886E+00 0.1181945E+00
 0.1019301E+00 0.8327674E-01 0.6267205E-01 0.4060143E-01 0.1761401E-01
 15 13  5  4  5  0  0  0  1  1
 0.2500000E+01 0.1000000E+01 0.2000000E+01 0.0000000E+00
  0.0000  0.2588  0.5000  0.7071  0.8660  0.9659  0.9914  1.0000  0.9914  0.9659
  0.8660  0.7071  0.5000  0.2588  0.0000
 -1.0000 -0.9659 -0.8660 -0.7071 -0.5000 -0.2588 -0.1305  0.0000  0.1305  0.2588
  0.5000  0.7071  0.8660  0.9659  1.0000
  0.0000  0.2415  0.4830  0.7244  0.9659  0.9914  1.0000  0.9914  0.9659  0.7244
  0.4830  0.2415  0.0000
 -0.2588 -0.2588 -0.2588 -0.2588 -0.2588 -0.1305  0.0000  0.1305  0.2588  0.2588
  0.2588  0.2588  0.2588
$STOP
/*
//
PRINTED OUTPUT
 NT NPHI
  2   20
XT
-0.5773503E+00 0.5773503E+00
AT
 0.1000000E+01 0.1000000E+01
X
-0.9931286E+00-0.9639719E+00-0.9122344E+00-0.8391170E+00-0.7463319E+00
-0.6360537E+00-0.5108670E+00-0.3737061E+00-0.2277859E+00-0.7652652E-01
 0.7652652E-01 0.2277859E+00 0.3737061E+00 0.5108670E+00 0.6360537E+00
 0.7463319E+00 0.8391170E+00 0.9122344E+00 0.9639719E+00 0.9931286E+00
A
 0.1761401E-01 0.4060143E-01 0.6267208E-01 0.8327675E-01 0.1019301E+00
 0.1181945E+00 0.1316886E+00 0.1420961E+00 0.1491730E+00 0.1527534E+00
 0.1527534E+00 0.1491730E+00 0.1420961E+00 0.1316886E+00 0.1181945E+00
 0.1019301E+00 0.8327675E-01 0.6267208E-01 0.4060143E-01 0.1761401E-01
 NA NB MA MB MC LA LB LC M1 M2
 15 13  5  4  5  0  0  0  1  1
      BK             UR             ER           THR(1)
 0.2500000E+01 0.1000000E+01 0.2000000E+01 0.0000000E+00
RA
  0.0000  0.2588  0.5000  0.7071  0.8660  0.9659  0.9914  1.0000  0.9914  0.9659
  0.8660  0.7071  0.5000  0.2588  0.0000
ZA
 -1.0000 -0.9659 -0.8660 -0.7071 -0.5000 -0.2588 -0.1305  0.0000  0.1305  0.2588
  0.5000  0.7071  0.8660  0.9659  1.0000
RB
```

```
 0.0000   0.2415   0.4830   0.7244   0.9659   0.9914   1.0000   0.9914   0.9659   0.7244
 0.4830   0.2415   0.0000
ZB
-0.2588 -0.2588 -0.2588 -0.2588 -0.2588 -0.1305   0.0000   0.1305   0.2588   0.2588
 0.2588   0.2588   0.2588
 1TH MODE ELECTRIC AND MAGNETIC CURRENTS

ELECTRIC CURRENT ON FIRST PART OF OUTSIDE CONDUCTOR

  REAL JT      IMAG JT      MAG JT
 0.6325E+00-0.8715E+00 0.1077E+01
 0.5889E+00-0.8223E-01 0.5946E+00
 0.2590E+00 0.7627E+00 0.8055E+00
-0.4205E+00 0.1188E+01 0.1261E+01

  REAL JP      IMAG JP      MAG JP
-0.6092E+00 0.1065E+01 0.1227E+01
-0.5386E+00 0.6909E+00 0.8760E+00
-0.3348E+00 0.4160E+00 0.5339E+00
-0.8079E-01 0.2966E+00 0.3074E+00
-0.5854E-01 0.5308E+00 0.5340E+00

ELECTRIC CURRENT ON SECOND PART OF OUTSIDE CONDUCTOR

  REAL JT      IMAG JT      MAG JT
-0.4426E+00-0.1791E+01 0.1844E+01
 0.4508E+00-0.1904E+01 0.1957E+01
 0.1105E+01-0.1607E+01 0.1950E+01
 0.1478E+01-0.1261E+01 0.1943E+01

  REAL JP      IMAG JP      MAG JP
-0.8943E+00-0.5344E+00 0.1042E+01
-0.2190E+00-0.1271E+01 0.1290E+01
 0.5212E+00-0.1483E+01 0.1572E+01
 0.1119E+01-0.1365E+01 0.1765E+01
 0.1525E+01-0.1183E+01 0.1930E+01

ELECTRIC CURRENT ON FIRST PART OF INSIDE CONDUCTOR

  REAL JT      IMAG JT      MAG JT
 0.1267E+01 0.3871E-01 0.1268E+01
 0.2608E+00 0.5877E-01 0.2673E+00
-0.7262E+00 0.1498E+00 0.7415E+00

  REAL JP      IMAG JP      MAG JP
-0.1558E+01-0.3328E-01 0.1558E+01
-0.1116E+01-0.6796E-01 0.1118E+01
-0.7062E+00-0.7621E-01 0.7103E+00
-0.1707E+00-0.3173E+00 0.3603E+00

ELECTRIC CURRENT ON SECOND PART OF INSIDE CONDUCTOR

  REAL JT      IMAG JT      MAG JT
-0.7549E+00-0.2661E+00 0.8005E+00
 0.2513E+00-0.4657E-01 0.2556E+00
 0.1280E+01 0.8836E-01 0.1283E+01

  REAL JP      IMAG JP      MAG JP
 0.6677E+00-0.1862E+00 0.6932E+00
 0.7989E+00 0.7428E-02 0.7989E+00
```

0.1129E+01	0.6751E-01	0.1131E+01
0.1584E+01	0.1214E+00	0.1589E+01

ELECTRIC CURRENT IN APERTURE

REAL JT	IMAG JT	MAG JT
-0.1102E+01	0.6770E+00	0.1294E+01
-0.1261E+01	0.3286E+00	0.1303E+01
-0.1339E+01	-0.4550E-01	0.1339E+01
-0.1305E+01	-0.4478E+00	0.1380E+01
-0.1196E+01	-0.9023E+00	0.1498E+01

REAL JP	IMAG JP	MAG JP
-0.1580E+00	0.2658E+00	0.3092E+00
-0.2728E+00	0.1534E+00	0.3130E+00
-0.3750E+00	0.5005E-01	0.3783E+00
-0.5571E+00	-0.5482E-01	0.5598E+00

MAGNETIC CURRENT IN APERTURE

REAL JT	IMAG JT	MAG JT
0.1429E+00	0.7546E-01	0.1616E+00
0.1589E+00	0.1393E+00	0.2113E+00
0.1049E+00	0.1579E+00	0.1896E+00

REAL JP	IMAG JP	MAG JP
0.6528E+00	0.1541E+00	0.6708E+00
0.1559E+00	0.3855E+00	0.4158E+00
-0.1000E+00	0.4523E+00	0.4633E+00
-0.8395E+00	0.4811E+00	0.9675E+00

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[3] J. R. Mautz and R. F. Harrington, "An Improved E-field Solution for a Conducting Body of Revolution," Report RADC-TR-80-194, Rome Air Development Center, Griffiss Air Force Base, New York, June 1980.

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